METHODS

FOR

EARTHWORK COMPUTATIONS

BY

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PREFACE.

An attempt has been made in this book to formulate a series of rules by which the terms necessary for the numerical computation of volumes, either by the prismoidal formula or by the average end area method, may be written directly from the notes. In each of the first three chapters a general rule, applicable to any case, is first stated; and then special rules, applicable to the cases that most frequently occur in practice, are given, these rules gaining in simplicity as their application becomes more limited.

Tables and diagrams have been issued in large numbers, each prepared for a special case and necessarily limited in extent, and many of them are valuable in their separate fields. But the slide rule has been coming into such general use in recent years that the author has deemed it advisable to prepare one, described in Chapter VI, that may be used for earthwork determinations. The adopted arrangement of the scales makes the instrument general in application, so much so that it will enable the computer to determine the volume in any case considered in this book.

The author wishes to call the reader's attention to Appendix III, in which the rules are stated in a manner that, it is believed, will make their application in practice simple and rapid, with little chance of error.

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METHODS FOR EARTHWORK COMPUTATIONS.

INTRODUCTION.

1. Purpose of Chapter I. — It is generally admitted that, as a rule, the use of the so-called prismoidal formula leads to a value of a volume more accurate than that obtained by any other approximate method in common use, but the labor involved is so great that it is customary to use a less exact method, usually the method of average end areas. The calculation of the areas of the two ends, and the determination of the dimensions and then of the area of the mid-section, with the subsequent combination of these to find the volume, indeed form a lengthy process, and the purpose of Chapter I is to explain a method by which the labor is greatly reduced.

Rule 1 of that chapter formulates a statement by which the terms involved in the computation may be written directly from the notes, without any intermediate steps and without drawing any figure, the symbolized form of the rule enabling the computer to apply it without difficulty; and Arts. 24 and 25 explain in full its application to the complicated solid shown in Fig. 17. Other rules in the same chapter provide simpler methods for special cases, but Rule 1 is the fundamental rule, applicable to the general solid.

The reader may test the general method, without preliminary preparation, by studying Arts. 16, 17, 18, 24 and 25, remembering that the problem was selected, not to demonstrate the ease with which the method can be used in some special case, but to explain the way in which the terms are formed in a solid that is by no means simple.

2. Chapter II contains a number of rules by which the computer can form in a systematic manner, directly from the notes, the terms that occur in the determination of a volume by the average end area method, Rule 14 being the general rule, applicable to any polygonal cross-section.

3. Chapter III determines, in certain cases, the correction to be subtracted algebraically from the volume computed by the average end area method in order to find the volume as it would be given by the prismoidal formula, the quantities being taken directly from the notes.

4. Chapter IV treats of the determination of the volume, by the average end area method, when the transverse slope of the surface is measured; and the resulting formulas are so systematized in Art. 124 that the volumes in side-hill work as well as in thorough work may be readily computed.

5. Chapter V is devoted to the consideration of the correction for curvature in railroad work.

6. Chapter VI describes the Crockett Volume Slide Rule, and explains the method of applying it to the determination of volumes.

7. Approximate Results of Earthwork Determination. — For two reasons the computation of the volume of earthwork is, at its best, merely approximate:

First. — The measurements, upon which the computation depends, are approximate and liable to error. Thus, when the measurements are made to the nearest tenth of a foot, any measurement is liable to an error of 0.05 foot; and the error, in the volume of a parallelopiped whose base is 14×100 feet and whose altitude may be in error by 0.05 foot, might be 70 cubic feet or 2.6 cubic yards.

Second. — The formulas are based upon the assumption that the surfaces are either plane surfaces or hyperbolic paraboloids, — or upon yet cruder assumptions, — and in practice this cannot be other than approximately true.

The degree of refinement to which the measurements should be made, the extent to which the computations should be carried

out, the cost of making the measurements and the computations, and the cost of the earthwork, are interdependent. Increased refinement in the measurements necessitates increased cost in securing them; the farther out the computations are carried, the more is the time required, and hence the greater the resulting expenditure; and the economy resulting from the increased accuracy must more than compensate for the added cost in order that it may be justified. The computations should not be carried out farther than is warranted by the measurements; the measurements should not be more delicate than is necessary to give the proper degree of accuracy to the computations; and both depend upon the cost of the earthwork.

Thus, in excavating rock at $1.00 per cubic yard, the measurements should be more carefully made and the computations carried out farther than would be the case in dealing with earth at 25 cents per cubic yard. On the other hand, it would not be profitable to pay the engineering corps $1.25 to gather additional information that would save only 4 cubic yards at 25 cents a yard.

In railway work, it is customary to make measurements to the nearest tenth of a foot. Such measurements do not justify carrying the computations to beyond the nearest cubic yard; in fact, it seems that any method which gives the volume with an error not exceeding 1 in 400 or 1 in 600 is sufficiently accurate.

CHAPTER I.

APPLICATION OF THE PRISMOIDAL FORMULA WHEN THE CROSS-SECTIONS ARE DETERMINED BY LEVELS.

8. The Prismoidal Formula. — It is shown in geometry that the volume of a solid, each end of which is a polygon, the end planes being parallel and the faces being either plane triangles or plane trapezoids, is given correctly by the prismoidal formula,

$$P = \tfrac{1}{6} L (A + A' + 4 A_m) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where A and A' are the areas of the two ends, A_m is the area of the section cut by a plane parallel to the two ends and midway between them, and L is the perpendicular distance between the two ends.

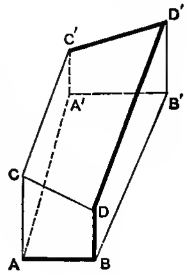

Fig. 1.

Eq. (1) will also give correctly the volume of any solid satisfying the following conditions:*

(*a*) The base $AA'B'B$ shall be in a horizontal plane.

(*b*) The sides $AA'C'C$ and $BB'D'D$ shall be in vertical planes.

(*c*) The ends $ABDC$ and $A'B'D'C'$ shall be in parallel vertical planes.

(*d*) The top $CDD'C'$ shall be generated by a straight line moving along the lines CC' and DD' and always parallel to the two end planes.

Hence the volume between the horizontal plane $EFF'E'$ (Fig. 2), and the surface † $ABCC'B'A'$ will be given correctly by Eq. (1); so also will the volume between $EFF'E'$ and $ADCC'D'A'$; and, since the subtraction of the latter from the former will give the volume $ADCB$-$A'D'C'B'$, it follows that the prismoidal formula will give the volume of $ADCB$-$A'D'C'B'$.

* See Appendix I.

† The surfaces $ABB'A'$, $BB'C'C$, $ADD'A'$ and $DD'C'C$ must satisfy the condition (*d*).

9. Description of the General Solid. — From Art. 8 we see that the prismoidal formula will give the correct volume of any solid which satisfies the following conditions:

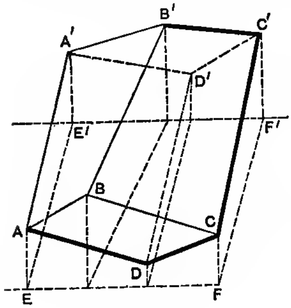

Fig. 2.

(*a*) Both ends shall be wholly in excavation, or both shall be wholly in embankment.

(*b*) The ends shall be in parallel planes. The end section may be a point, a line, or any polygon bounded by straight lines.

(*c*) The longitudinal edges shall be straight lines joining the vertices of the two ends.

(*d*) Each longitudinal face shall be generated by a straight line moving along the two bounding longitudinal edges and always parallel to the planes of the ends. Hence the faces are either plane surfaces or portions of hyperbolic paraboloids.

10. Determination of the End Sections. — The vertices of each end section may be located in different ways, the usual method being to measure the elevation of each vertex above (+), or its depression below (−), a given horizontal line, and its horizontal distance to the right (+), or left (−), of a given vertical line, these lines being in the plane of the end in which the vertex lies.

The horizontal lines need not be in the same horizontal plane, nor need the plane through the two vertical lines be at right angles to the end planes, but the length L of the solid must be the perpendicular distance between the end planes.

11. Application to Practice. — In determining the volume of a solid whose surfaces differ from those described in Art. 9, it should be divided by parallel cross-sections into a number of shorter solids, each one of which should approximate as nearly as may be to the description of the general solid.

12. Illustrative Solid. — A solid corresponding to these conditions is shown in Fig. 3, where $ABCDEFGHIJK$ is one end, $A'B'E'F'G'H'J'L'$ the other, OX and OY the reference lines at

one end, and $O'X'$ and $O'Y'$ those at the other, the planes XOY and $X'O'Y'$ of the ends being parallel. The surfaces $L'AA'$, $BB'C$, $CB'D$, $HH'I$, and $JJ'K$ are plane triangles; and the others, including the surface $KJ'L'A$, are hyperbolic paraboloids.

The base plane OX-$O'X'$ may be inclined to a horizontal plane, but *the length of the solid must always be the perpendicular distance between its ends.*

If vertical planes are passed through the longitudinal edges in Fig. 3, and if we consider the volumes above the plane OX-$O'X'$,

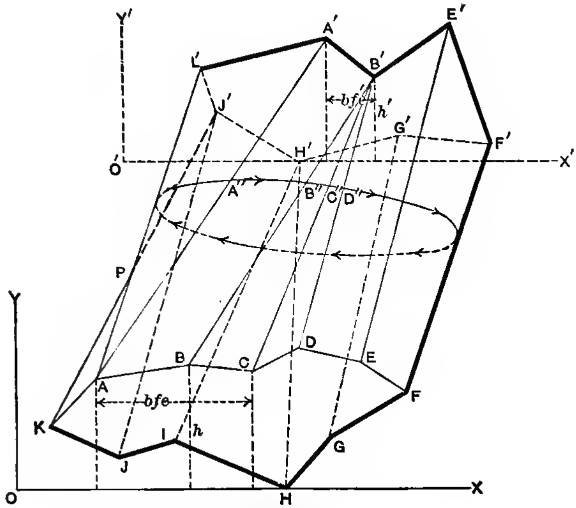

Fig. 3.

we see that the volume of the solid is equal to the sum of the volumes below the six surfaces $L'AA'$, $AA'B'B$, $BB'C$, $CB'D$, $DB'E'E$ and $EE'F'F$ diminished by the sum of the volumes below the five surfaces $FF'G'G$, $GG'H'H$, $HH'I$, $IH'J'J$ and $JJ'K$, and increased by the volume below KPA, and decreased by that below $J'PL'$.*

13. The Method of this Chapter consists in first applying the prismoidal formula to the solid shown in Fig. 4, then determining the modifications of the result in special cases, and finally apply-

* See Art. 4 of Appendix I.

ing these results to the derivation of general formulas and the rules by which these formulas may be written.

If the measurements are made in feet, the volumes will be in cubic feet, and the results must be divided by 27 to reduce them to cubic yards.

14. The Prismoidal Formula Applied. — The General Case. — The ends being trapezoids, their areas are

$$A = \tfrac{1}{2} m (h_1 + h_2), \quad A' = \tfrac{1}{2} m' (h_1' + h_2').$$

The mid-section is a trapezoid, whose parallel vertical sides are $\frac{1}{2}(h_1 + h_1')$ and $\frac{1}{2}(h_2 + h_2')$, the horizontal distance between them being $\frac{1}{2}(m + m')$; hence its area is

$$A_m = \tfrac{1}{2} \frac{m + m'}{2} \left(\frac{h_1 + h_1'}{2} + \frac{h_2 + h_2'}{2} \right)$$

$$= \tfrac{1}{8} (m + m')(h_1 + h_2 + h_1' + h_2').$$

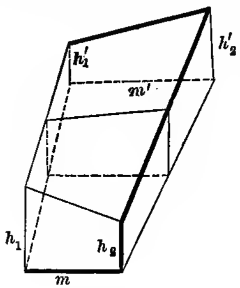

Fig. 4.

Therefore the volume by the prismoidal formula is, in cubic feet,

$$P = \tfrac{1}{6} L [\tfrac{1}{2} m (h_1 + h_2) + \tfrac{1}{2} m'(h_1' + h_2')$$
$$+ \tfrac{1}{2}(m + m')(h_1 + h_2 + h_1' + h_2')].$$

$$\therefore \boldsymbol{P = \tfrac{1}{12} L [(2 h_1 + h_1') m + (2 h_1' + h_1) m'}$$
$$\boldsymbol{+ (2 h_2 + h_2') m + (2 h_2' + h_2) m']} \quad . \quad (2)$$

The quantity in the brackets in Eq. (2) may be written by using the rule: — *For each longitudinal top edge: At each end, multiply twice the height at that end + the height at the other end of the edge by the horizontal width at the first end; and add the four results.*

NOTE 1. — Eq. (2) may be written

$$P = \tfrac{1}{12} L [(h_1 + h_2)(2 m + m') + (h_1' + h_2')(2 m' + m)],$$

and the quantity in the brackets is given by the rule: — *At each end, multiply the sum of the heights at that end by twice the width at that end + the width at the other end; and add the two results.*

NOTE 2.— If a vertex in one end-section, as A, is below the line OX (Fig. 5), the distance aA must be considered negative. Eq. (2) will then give the *difference* between the volume above $CDa'b'b$ and that below aDC, negative when the latter exceeds the former. Hence, in finding the volume of a solid, it will be simpler if both the lines OX and $O'X'$ are above the end sections, or both below, so that both ends may be wholly in excavation, or both wholly in embankment.

NOTE 3.— If the point B' is to the left of A' (Fig. 6), so that the edges AA' and BB', as seen from above, appear to cross each other at P, the dis-

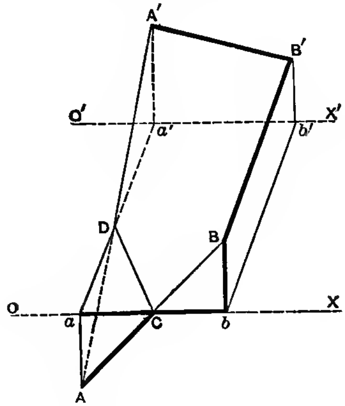

Fig. 5.

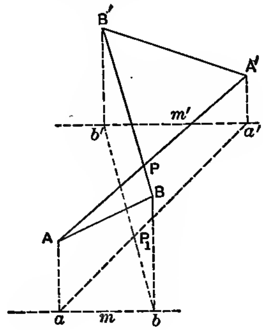

Fig. 6.

tance m' must be considered negative. Eq. (2) then gives the *difference* between the volumes above aP_1b and $a'P_1b'$, negative when the latter exceeds the former.*

Fig. 4 will assume different forms according to the values assigned to m, h_1, h_2 and m', h_1', h_2', but Eq. (2) will in each case give the volume correctly. Some of these special cases are considered in the next article.

15. The Prismoidal Formula Applied. — Special Cases. — We shall consider six cases.

(1) *The top a plane or a warped surface* †, *and both end sections right triangles*, Figs. 7 and 8. In Fig. 7, $h_1 = h$, $h_2 = 0$, $h_1' = h'$, $h_2' = 0$, and hence by Eq. (2) or by the rule,

$$P = \tfrac{1}{12} L\,[(2\,h + h')\,m + (2\,h' + h)\,m'] \quad . \quad . \quad . \qquad (3)$$

* If an insect on top of the surface APB moves along the surface until it is near $B'A'$, it will be below $B'PA'$.

† Hyperbolic paraboloid.

In Fig. 8, $h_1 = 0$, $h_2 = h$, $h_1' = h'$, $h_2' = 0$, and hence

$$P = \tfrac{1}{12} L\,(h'm + 2\,h'm' + 2\,hm + hm')$$
$$= \tfrac{1}{12} L\,[(2\,h + h')\,m + (2\,h' + h)\,m'] \quad . \quad . \quad . \quad (3)$$

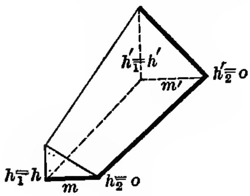

Fig. 7.

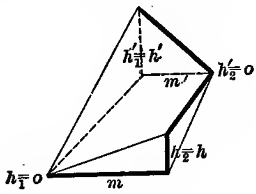

Fig. 8.

(2) *The top a warped surface,* one end a triangle and the other a vertical line,* Fig. 9.

$$P = \tfrac{1}{12} L\,(2\,h + h')\,m \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

(3) *The top a plane surface, one end a trapezoid, and the other a*

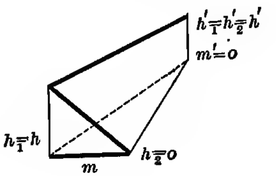

Fig. 9.

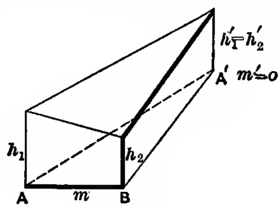

Fig. 10.

vertical line (a wedge or a truncated vertical triangular prism), Fig. 10.

$$P = \tfrac{1}{12} L\,[(2\,h_1 + h_1')\,m + (2\,h_2 + h_2')\,m] \quad . \quad . \quad (5)$$
$$= \tfrac{1}{12} L\,[2\,m\,(h_1 + h_2 + h_1')]$$
$$= \tfrac{1}{2} Lm\,\frac{h_1 + h_2 + h_1'}{3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

But the area of the base ABA' is $\frac{1}{2}\,Lm$, so that from Eq. (6) the volume of a truncated triangular prism is

$$\boldsymbol{P} = \textbf{area of base} \times \tfrac{1}{3}\ \textbf{the sum of the vertical edges.}$$

* Hyperbolic paraboloid.

(4) *The top a plane surface, one end a trapezoid, and the other a point,* Fig. 11.

$$P = \tfrac{1}{12} L\,(2\,h_1 m + 2\,h_2 m) \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$
$$= \tfrac{1}{2}\, Lm \frac{h_1 + h_2}{3}. \quad \text{See Eq. (6).}$$

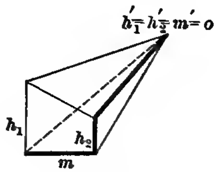

Fig. 11.

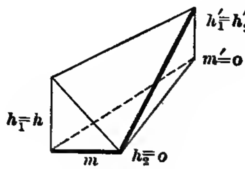

Fig. 12.

(5) *The top a plane surface, one end a right triangle, and the other a vertical line,* Fig. 12.

$$P = \tfrac{1}{12} L\,[(2\,h + h_1')\,m + h_2' m] \quad . \quad . \quad . \quad . \quad . \quad (8)$$
$$= \tfrac{1}{12} L\,(2\,h + 2\,h')\,m$$
$$= \tfrac{1}{2}\, Lm \frac{h + h'}{3}. \quad \text{See Eq. (6).}$$

(6) *The top a plane surface, both end sections right triangles,* Fig. 13. In this case the angle θ is the same at the two ends. From Eq. (2),

$$P = \tfrac{1}{12} L\,[(2\,h + h')\,m + (2h' + h)\,m']$$
$$= \tfrac{1}{12} L\,(2\,hm + h'm + 2\,h'm' + hm').$$

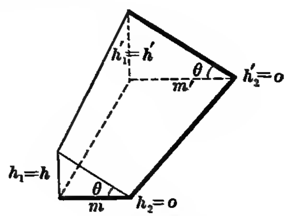

Fig. 13.

But $\dfrac{h'}{m'} = \dfrac{h}{m}$, so that $h'm = hm'$, and therefore

$$P = \tfrac{1}{12} L\,[2\,(h + h')\,m + 2\,h'm'] \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

or

$$P = \tfrac{1}{12} L\,[2\,hm + 2\,(h' + h)\,m'] \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

THE GENERAL RULE.

16. Definitions. — If we place a pencil-point at A'' on the edge AA' in Fig. 3, and then move the pencil clockwise around the solid (from A'' to B'' to C'' to D'' and so on), it will come in

succession to the edges AA', BB', CB', DB', . . . IH', JJ', KJ', AL'. Of any three consecutive edges AA', BB', CB', we shall call BB' the **middle edge,** CB' the **forward edge,** and AA' the **back edge.**

We shall show that, in the expression for the volume of the solid shown in Fig. 3, the terms involving the heights at the ends of any longitudinal edge may be written, in each of the four possible cases, by the use of the following rule:

17. The General Rule for Prismoidal Volumes. Rule 1. — *For each longitudinal edge: At each end, to twice the height at that end add the height at the other end, and multiply the sum by the horizontal distance between the back and forward edges at the first end (negative when that end of the forward edge is to the left of the corresponding end of the back edge), and multiply the product by the length of the solid* ÷ 12. *Add the results algebraically.*

The sign of the term may be obtained by subtracting, algebraically, the horizontal distance from the vertical line OY (or $O'Y'$) to the height at the end of the back edge from the horizontal distance to the height at the corresponding end of the forward edge, distances to the right of OY (or $O'Y'$) being considered positive, and those to the left negative.

Note that, corresponding to each edge, there are two terms, one of which may equal zero. If an edge lies in the base plane OX-$O'X'$, the height at each end will be zero, and both the corresponding terms will be equal to zero.

If h and h' are the heights at the ends of any longitudinal edge, and bfe and bfe' the horizontal distances between the back and forward edges at the two ends respectively, Rule 1 may be symbolized as follows:

18. Rule 1. The General Solid. — Clockwise, for each edge,

$$\tfrac{1}{12} L\,(2\,h + h')\,bfe \qquad (- \text{ when } f \text{ is left of } b),$$

$$\tfrac{1}{12} L\,(2\,h' + h)\,bfe' \qquad (- \text{ when } f' \text{ is left of } b').$$

19. Case 1. — In Fig. 14, which shows three adjacent edges AA', BB' and CC' of a surface,* the volume consists of two parts,

* In Figs. 14, 15 and 16, the ends are in parallel planes.

that above $abb'a'$, and that above $bcc'b'$. By Eq. (2) we have for the total volume,

$$\begin{aligned}P &= \tfrac{1}{12} L\,[(2\,h_1 + h_1')\,m_1 + (2\,h_1' + h_1)\,m_1' + (2\,h_2 + h_2')\,m_1 \\ &\qquad + (2\,h_2' + h_2)\,m_1'] \\ &+ \tfrac{1}{12} L\,[(2\,h_2 + h_2')\,m_2 + (2\,h_2' + h_2)\,m_2' + (2\,h_3 + h_3')m_2 \\ &\qquad + (2\,h_3' + h_3)\,m_2'] \\ &= \tfrac{1}{12} L\,[(2\,h_1 + h_1')\,m_1 + (2\,h_1' + h_1)\,m_1'] \\ &+ \tfrac{1}{12} L\,[(2\,h_2 + h_2')\,(m_1 + m_2) + (2\,h_2' + h_2)\,(m_1' + m_2')] \\ &+ \tfrac{1}{12} L\,[(2\,h_3 + h_3')\,m_2 + (2\,h_3' + h_3)\,m_2'],\end{aligned}$$

so that the terms containing the heights h_2 and h_2' at the ends of the middle longitudinal edge are

$$\tfrac{1}{12} L\,(2\,h_2 + h_2')\,(m_1 + m_2) \text{ and } \tfrac{1}{12} L\,(2\,h_2' + h_2)\,(m_1' + m_2')$$

in accordance with Rule 1.

20. Case 2. — In finding the volume $ABCDE\text{-}A'B'C'D'E'$ (Fig. 15), we would find the volume under the surface $ABCC'B'A'$ and from it subtract the volume under $EDCC'D'E'$.

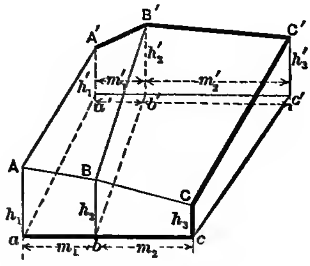

Fig. 14.

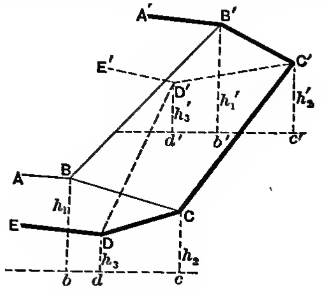

Fig. 15.

Considering only the three edges BB', CC' and DD', of which CC' is the middle edge, we have for the volume under $BCC'B'$, by Eq. (2),

$$\begin{aligned}&\tfrac{1}{12} L\,[(2\,h_1 + h_1')\,\overline{bc} + (2\,h_1' + h_1)\,\overline{b'c'} + (2\,h_2 + h_2')\,\overline{bc} \\ &\qquad + (2\,h_2' + h_2)\,\overline{b'c'}],\end{aligned}$$

and for that under $DCC'D'$,

$$\begin{aligned}&\tfrac{1}{12} L\,[(2\,h_3 + h_3')\,\overline{dc} + (2\,h_3' + h_3)\,\overline{d'c'} + (2\,h_2 + h_2')\,\overline{dc} \\ &\qquad + (2\,h_2' + h_2)\,\overline{d'c'}].\end{aligned}$$

Subtracting the latter from the former, we have

$$\tfrac{1}{12} L\,[(2\,h_1 + h_1')\,\overline{bc} + (2\,h_1' + h_1)\,\overline{b'c'}]$$
$$+ \tfrac{1}{12} L\,[(2\,h_2 + h_2')\,(\overline{bc} - \overline{dc}) + (2\,h_2' + h_2)\,(\overline{b'c'} - \overline{d'c'})]$$
$$- \tfrac{1}{12} L\,[(2\,h_3 + h_3')\,\overline{dc} + (2\,h_3' + h_3)\,\overline{d'c'}],$$

so that the terms in the required volume that contain the middle heights h_2 and h_2' are

$$\tfrac{1}{12} L\,(2\,h_2 + h_2')\,\overline{bd} \text{ and } - \tfrac{1}{12} L\,(2\,h_2' + h_2)\,\overline{d'b'}.$$

These terms may be found by Rule 1, the result being positive when the forward height is to the right of the back height (as in the end *BCD*), and negative when the forward height is to the left of the back height (as in the end *B'C'D'*).

21. Case 3. — If, in Fig. 14, *ABCC'B'A'* is the lower surface of a solid, the volume below this surface should be subtracted from the volume below the upper surface of the solid. But the line *AA'* will now be the forward edge, and *CC'* the back edge, the forward edge being to the left of the back edge; so that, if we give the terms the positive sign when the forward height is to the right of the back height, and the negative when it is to the left, the terms will have their proper signs for algebraic addition.

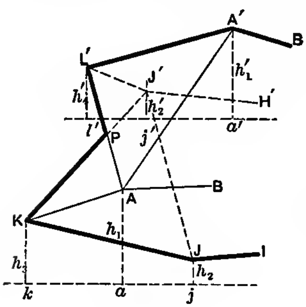

Fig. 16.

22. Case 4. — The peculiarity in Fig. 16 is the presence of the warped surface *KJ'L'A*.

The volume *IJKAB-H'J'L'A'B'* equals that below the surfaces *BAA'B'*, *AA'L'* and *AKP*, diminished by that below *IJJ'H'*, *JJ'K* and *L'PJ'*. Considering only the surfaces of which either *AL'* or *KJ'* is a border line, we find by Eq. (2) that the volume below *AA'L'* is

$$\tfrac{1}{12} L\,[(2\,h_4' + h_1)\,\overline{l'a'} + (2\,h_1' + h_1)\,\overline{l'a'}];$$

and the volume below *JJ'K* is

$$\tfrac{1}{12} L\,[(2\,h_3 + h_2')\,\overline{kj} + (2\,h_2 + h_2')\,\overline{kj}];$$

and the volume below AKP diminished by that below $L'PJ'$ is *

$$\tfrac{1}{12} L\,[(2\,h_3 + h_2')\,\overline{ka} + (2\,h_2' + h_3)\,(-\,\overline{l'j'}) + (2\,h_1 + h_4')\,\overline{ka} + (2\,h_4' + h_1)\,(-\,\overline{l'j'})].$$

Subtracting the second from the sum of the first and third, we have

$$\tfrac{1}{12} L\,(2\,h_1' + h_1)\,\overline{l'a'} - \tfrac{1}{12} L\,(2\,h_2 + h_2')\,\overline{kj}$$
$$+ \tfrac{1}{12} L\,[(2\,h_3 + h_2')\,(\overline{ka} - \overline{kj}) + (2\,h_2' + h_3)\,(-\,\overline{l'j'})]$$
$$+ \tfrac{1}{12} L\,[(2\,h_1 + h_4')\,\overline{ka} + (2\,h_4' + h_1)\,(\overline{l'a'} - \overline{l'j'})],$$

so that the terms corresponding to KJ' as the middle edge are

$$-\,\tfrac{1}{12} L\,(2\,h_3 + h_2')\,\overline{aj} \text{ and } -\,\tfrac{1}{12} L\,(2\,h_2' + h_3)\,\overline{l'j'},$$

and those corresponding to AL' as the middle edge are

$$\tfrac{1}{12} L\,(2\,h_1 + h_4')\,\overline{ka} \text{ and } \tfrac{1}{12} L\,(2\,h_4' + h_1)\,\overline{j'a'},$$

which may be found by Rule 1, the resulting term being given the negative sign when the forward height is to the left of the back height.

23. In each of the Four Cases, therefore, the terms corresponding to each longitudinal edge are given by the General Rule,— Rule 1, — and therefore this rule will determine the volume of any solid that satisfies the conditions specified in Art. 9.

24. Example. — In Fig. 17, let $ABCDEE'D'C'A'$ be the original surface of the ground, and $AHGFEE'F'G'A'$ the surface after the volume

$ABCDEFGH\text{-}A'C'D'E'F'G'$

has been removed. We wish to determine the number of cubic yards in this volume.

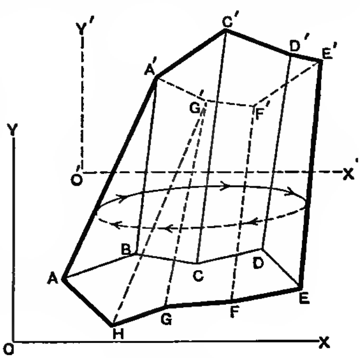

Fig. 17.

* See Note 3 in Art. 14.

Let us assume that the solid is 60 feet long, and that the notes for the original surface are

$$\begin{array}{lccccc}
 & \multicolumn{2}{c}{A'} & C' & D' & E' \\
A'C'D'E' & \multicolumn{2}{c}{\dfrac{10}{+50}} & \dfrac{40}{+60} & \dfrac{58}{+52} & \dfrac{74}{+55} \\
 & / & \backslash & | & | & | \\
ABCDE & \dfrac{20}{+30} & \dfrac{40}{+40} & \dfrac{60}{+34} & \dfrac{80}{+50} & \dfrac{90}{+35} \\
 & A & B & C & D & E
\end{array}$$

and those for the new surface are

$$\begin{array}{lccccc}
 & A' & \multicolumn{2}{c}{G'} & F' & E' \\
A'G'F'E' & \dfrac{10}{+50} & \multicolumn{2}{c}{\dfrac{30}{+35}} & \dfrac{45}{+30} & \dfrac{74}{+55} \\
 & | & / & \backslash & | & | \\
AHGFE & \dfrac{20}{+30} & \dfrac{30}{+10} & \dfrac{50}{+18} & \dfrac{70}{+20} & \dfrac{90}{+35} \\
 & A & H & G & F & E
\end{array}$$

where the horizontal distances from the lines OY and $O'Y'$ respectively are in the numerators, and the vertical distances above (+) the lines OX and $O'X'$ respectively are in the denominators of the fractional expressions, and the slanting lines indicate the longitudinal surface edges of the solid.

To avoid the necessity of drawing the figure, arrange these notes * so that the points shall be given in clockwise order, as shown by the arrowheads in Fig. 17, repeating the first two edges:

$$\begin{array}{cccccccccc}
\multicolumn{2}{c}{A'} & C' & D' & E' & F' & \multicolumn{2}{c}{G'} & \multicolumn{2}{c}{A'} \\
\multicolumn{2}{c}{\dfrac{10}{+50}} & \dfrac{40}{+60} & \dfrac{58}{+52} & \dfrac{74}{+55} & \dfrac{45}{+30} & \multicolumn{2}{c}{\dfrac{30}{+35}} & \multicolumn{2}{c}{\dfrac{10}{+50}} \\
\cdot^{\cdot^{\cdot}} & \backslash & | & | & | & | & / & \backslash & / & \ddots \\
\dfrac{20}{+30} & \dfrac{40}{+40} & \dfrac{60}{+34} & \dfrac{80}{+50} & \dfrac{90}{+35} & \dfrac{70}{+20} & \dfrac{50}{+18} & \dfrac{30}{+10} & \dfrac{20}{+30} & \dfrac{40}{+40} \\
A & B & C & D & E & F & G & H & A & B
\end{array}$$

where the dotted lines are duplicates of others, and are inserted only for convenience in forming the terms.

* The original notes may be used without rearranging, after the method is understood.

Then the terms, in the formula for the volume, corresponding to each longitudinal edge, are found by Rule 1 as follows, the factor $\frac{1}{12} L$, or $\frac{1}{12} \times 60$, being the same for all the terms:

Place a pencil-point on the line BA' (in the notes); then, starting with the end B, twice the height at that end + the height at the other end $= 2h + h' = 2 \times 40 + 50 = 130$; the forward edge is CC' and the back edge is AA', so that bfe is the horizontal distance from A to $C = 60 - 20 = 40$; hence the term is

$$\tfrac{1}{12} \times 60 \times 130 \times 40.$$

At the end A', twice the height at that end + the height at the other end $= 2h' + h = 2 \times 50 + 40 = 140$; bfe', at this end, is the horizontal distance from A' to $C' = 40 - 10 = 30$; hence the term is

$$\tfrac{1}{12} \times 60 \times 140 \times 30.$$

As another illustration, place the pencil-point on GG'; starting at the end G, twice the height at that end + the height at the other end $= 2h + h' = 2 \times 18 + 35 = 71$; bfe, at this end, is the horizontal distance between F and $H = 70 - 30 = 40$, which should be given the negative sign since H is to the left of F;* hence the term is

$$\tfrac{1}{12} \times 60 \times 71 \times (-40).$$

At the end G', twice the height + the height at the other end $= 2h' + h = 2 \times 35 + 18 = 88$; bfe' at this end is the distance between F' and G' (since the adjacent edges are FF' and HG') $= 45 - 30 = 15$, which should be negative since G' is to the left of F'; hence the term is

$$\tfrac{1}{12} \times 60 \times 88 \times (-15).$$

The terms, in the volume, corresponding to each longitudinal edge, are found in a similar manner, with the following results, after dividing by 27 to reduce to cubic yards:

* Shown in the notes by the fact that $30 < 70$.

Edge.	Terms.	+ Vols.	− Vols.
BA′	$\frac{1}{324} \times 60 \times 130 \times 40$	$962\frac{26}{27}$	
	“ $\times 140 \times 30$*	$777\frac{21}{27}$	
CC′	“ $\times 128 \times 40$	$948\frac{4}{27}$	
	“ $\times 154 \times 48$	$1368\frac{24}{27}$	
DD′	“ $\times 152 \times 30$	$844\frac{12}{27}$	
	“ $\times 154 \times 34$	$969\frac{17}{27}$	
EE′	“ $\times 125 \times (-10)$†		$231\frac{13}{27}$
	“ $\times 145 \times (-13)$†		$349\frac{2}{27}$
FF′	“ $\times 70 \times (-40)$†		$518\frac{14}{27}$
	“ $\times 80 \times (-44)$†		$651\frac{23}{27}$
GG′	“ $\times 71 \times (-40)$†		$525\frac{25}{27}$
	“ $\times 88 \times (-15)$‡		$244\frac{12}{27}$
HG′	“ $\times 55 \times (-30)$†		$305\frac{15}{27}$
	“ $\times 80 \times (-20)$†		$296\frac{8}{27}$
AA′	“ $\times 110 \times 10$	$203\frac{19}{27}$	
	“ $\times 130 \times (-20)$†		$481\frac{13}{27}$
	Totals =	$6075\frac{15}{27}$	$3604\frac{17}{27}$
		$3604\frac{17}{27}$	
	∴ Volume =	$2470\frac{25}{27}$ cu. yds.	

Since the factor $\frac{60}{324}$ is common to all the terms, it would evidently be easier, if the volume were to be determined by the methods of arithmetic, to form the algebraic sum of the products of the other two factors, and then multiply this sum by $\frac{60}{324}$. But by the use of the Crockett Volume Slide Rule the value of the expressions

$$\frac{1}{324} L\,(2\,h + h')\,bfe \text{ and } \frac{1}{324} L\,(2\,h' + h)\,bfe'$$

may be determined mechanically, and hence the author has adopted the arrangement in the text.

25. Example. Second Method. — The volume may also be found by first computing the volume between the plane *OX-O′X′* and the upper surface, bounded by the vertical ends and by the

* The back edge is *AA′*, so that $bfe' = 40 - 10$.

† The end of the forward edge is to the left of the corresponding end of the back edge.

‡ The forward edge is *HG′*, so that $bfe' = 30 - 45$.

vertical planes $A_1AA'A_1'$ and $E_1EE'E_1'$; subsequently computing the volume between OX-$O'X'$ and the new surface, limited in the same way; and subtracting the second result from the first. Each of these solids will have two edges, A_1A_1' and E_1E_1', in OX-$O'X'$; hence, prefix and affix to the notes new fractions in which the denominators shall be zero and the numerators shall be the horizontal distances to A_1 and A_1' and to E_1 and E_1'. Hence, for the first solid,

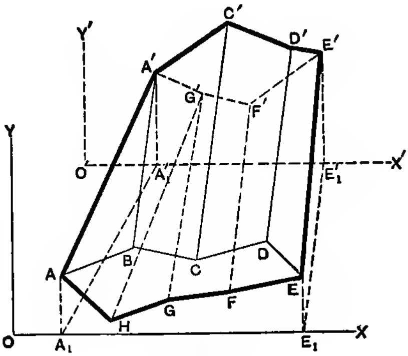

Fig. 17a.

	A_1'	A'		C'	D'	E'	E_1'
$A'C'D'E'$	$\frac{10}{0}$	$\frac{10}{+50}$		$\frac{40}{+60}$	$\frac{58}{+52}$	$\frac{74}{+55}$	$\frac{74}{0}$
$ABCDE$	$\frac{20}{0}$	$\frac{20}{+30}$	$\frac{40}{+40}$	$\frac{60}{+34}$	$\frac{80}{+50}$	$\frac{90}{+35}$	$\frac{90}{0}$
	A_1	A	B	C	D	E	E_1

and for the second solid,

	A_1'	A'	G'		F'	E'	E_1'
$A'G'F'E'$	$\frac{10}{0}$	$\frac{10}{+50}$	$\frac{30}{+35}$		$\frac{45}{+30}$	$\frac{74}{+55}$	$\frac{74}{0}$
$AHGFE$	$\frac{20}{0}$	$\frac{20}{+30}$	$\frac{30}{+10}$	$\frac{50}{+18}$	$\frac{70}{+20}$	$\frac{90}{+35}$	$\frac{90}{0}$
	A_1	A	H	G	F	E	E_1

When an edge lies in the plane OX-$O'X'$, both its h and its h' are zero, so that the terms corresponding to that edge are zero, and therefore should be neglected. Hence,

For the upper surface:

Edge.	Terms.	Vols.
AA'	$\frac{60}{324} \times 110 \times 20$	$407\frac{11}{27}$
	" $\times 130 \times 0$	0
BA'	" $\times 130 \times 40$	$962\frac{26}{27}$
	" $\times 140 \times 30$	$777\frac{21}{27}$
CC'	" $\times 128 \times 40$	$948\frac{4}{27}$
	" $\times 154 \times 48$	$1368\frac{24}{27}$
DD'	" $\times 152 \times 30$	$844\frac{12}{27}$
	" $\times 154 \times 34$	$969\frac{17}{27}$
EE'	" $\times 125 \times 10$	$231\frac{13}{27}$
	" $\times 145 \times 16$	$429\frac{17}{27}$
	Total =	$6940\frac{10}{27}$

For the lower surface:

Edge.	Terms.	Vols.
AA'	$\frac{60}{324} \times 110 \times 10$	$203\frac{19}{27}$
	" $\times 130 \times 20$	$481\frac{13}{27}$
HG'	" $\times 55 \times 30$	$305\frac{15}{27}$
	" $\times 80 \times 20$	$296\frac{8}{27}$
GG'	" $\times 71 \times 40$	$525\frac{25}{27}$
	" $\times 88 \times 15$	$244\frac{12}{27}$
FF'	" $\times 70 \times 40$	$518\frac{14}{27}$
	" $\times 80 \times 44$	$651\frac{23}{27}$
EE'	" $\times 125 \times 20$	$462\frac{26}{27}$
	" $\times 145 \times 29$	$778\frac{19}{27}$
	Total =	$4469\frac{12}{27}$

$$6940\tfrac{10}{27} - 4469\tfrac{12}{27}$$

Net volume = $2470\frac{25}{27}$ cu. yds.

26. Modifications of the General Rule. — Special forms of Rule 1 may be obtained for different cases, and we shall consider a few that are of practical value. Whenever the surface contains overhanging cliffs, Rule 1 should be used.

THE GENERAL RAILROAD SOLID RULE.

27. Railroad Cross-Sections. — Fig. 18 represents a railroad cross-section in an excavation, or cut, and Fig. 19 represents another in an embankment, or fill. Since Fig. 18 when inverted is essentially the same as Fig. 19, it is evident that formulas

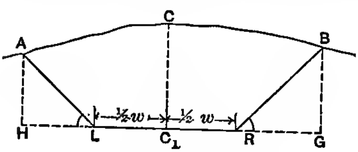

Fig. 18.

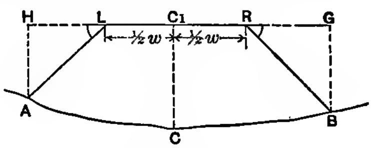

Fig. 19.

derived for the volume of an excavation are the same as those for an embankment.

In each figure, the horizontal line LR is the road-bed with width w (greater in a cut than in a fill, to allow for ditches); C_1 is the center of the road-bed; LA and RB are the side slopes, usually making the same angle $ALH = BRG$ with the road-bed, this angle being dependent upon the nature of the earth through which the cut passes, or of which the embankment is made; and

ACB is the surface of the ground, *A* and *B* being the points where the side slopes intersect the surface.

The surface *ACB* is located by the heights of the three points *A*, *C* and *B* (and of as many other points as may be deemed necessary) above * the line *HG*, and by their horizontal distances to the right or left of the vertical line C_1C through the center C_1 of the road-bed *LR*.

It is customary to write all the measurements for one section in the same horizontal line, the values for the left point *A* being at the left of the line, and the others being given in order towards the right. To save space, the measurements are written in a fractional form, the horizontal distance of the point from the center line C_1C being in the numerator, and its elevation above (or depression below) the road-bed *LR* in the denominator, a positive sign being prefixed for an elevation, and a negative for a depression.† On the left of the records there should be a column containing the numbers of the stations and other points where sections are taken, commencing at the bottom of the page and reading up, so that there may be no danger of confusion as to the right or left of the line. The engineer should also draw lines, in the notes, joining the points in adjacent cross-sections, to indicate the lines which in his judgment lie in the surface of the ground.

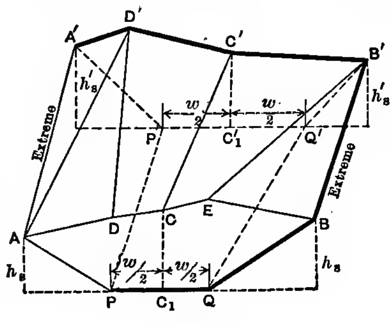

Fig. 20.

28. The Railroad Solid, or Prismoid as it is called, illustrated in Fig. 20 for an excavation, is a special case of the general solid, and its volume may always be found by Rule 1, the plane of the road-bed $PQQ'P'$ corresponding to the plane OX-$O'X'$ in Fig. 3; but Rule 2 is more easily applied.

Let us call the edges AA' and BB', in which the side slopes

* Below, for embankments.

† The fraction may be written in the inverse form,— elevations in the numerators, and horizontal distances in the denominators.

$PAA'P'$ and $QBB'Q'$ intersect the surface, the **extreme edges** of the surface $ADCEBB'C'D'A'$.

The terms, in the expression for the volume, corresponding to any longitudinal edge other than the extreme edges, are the same as those for any edge in the general solid.

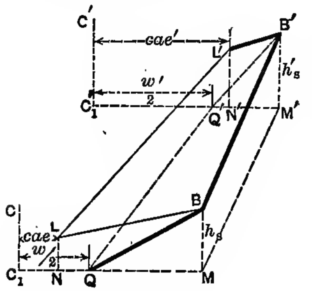

Fig. 21.

Since the edge QQ' (Fig. 20) is in the plane $PQQ'P'$, the height at each end is zero, and therefore the corresponding terms, in the formula for the volume, are each zero. That is, *if an edge lies in the reference plane, the terms corresponding to it, being zero, need not be written.*

The terms, in the expression for the volume, corresponding to the extreme edge BB' (Fig. 21), are, by Rule 1,

$$\tfrac{1}{12} L\,(2\,h_s + h_s')\,NQ \quad \text{and} \quad -\ \tfrac{1}{12} L\,(2\,h_s' + h_s)\,Q'N',$$

or $$\tfrac{1}{12} L\,(2\,h_s + h_s')(C_1Q - C_1N)$$

and $$+\tfrac{1}{12} L\,(2\,h_s' + h_s)\,(C_1'Q' - C_1'N'),$$

or $\tfrac{1}{12} L\,(2\,h_s + h_s')(\tfrac{1}{2}\,w - C_1N)$ and $+\tfrac{1}{12} L\,(2\,h_s' + h_s)\,(\tfrac{1}{2}\,w' - C_1'N')$.

These results may be written by the use of Rule 2.

29. The General Railroad Rule. Rule 2.—(*a*) *For each longitudinal edge, except the extreme edges: At each end, to twice the height at that end add the height at the other end, and multiply the sum by the horizontal distance between the adjacent longitudinal edges at the first end, and multiply the product by the length of the solid* ÷ 12.

(*b*) *For each extreme edge: At each end, to twice the height at that end add the height at the other end, and multiply the sum by the value, at that end, of* $\frac{1}{2}\,w$ * — *the horizontal distance* † *from the center to the adjacent edge, and multiply the product by the length of the solid* ÷ 12.

(*c*) *Add algebraically the results found by* (*a*) *and* (*b*).

* $\frac{1}{2}\,w$ is the horizontal distance, at the first end, from the center to that edge of the road-bed which is nearer the extreme edge.

† If the end of the adjacent edge is on the opposite side of the center line C_1C_1' from the corresponding end of the extreme edge, this distance must be considered negative.

Using the notation given in Appendix III, Rule 2 may be symbolized as follows:

30. Rule 2. The General Railroad Solid.

For each edge, except the extreme edges,

$$\tfrac{1}{12} L\,(2\,h + h')\,bfe,$$
$$\tfrac{1}{12} L\,(2\,h + h)\,bfe'.$$

For each extreme edge,

$$\tfrac{1}{12} L\,(2\,h_s + h_s')\,(\tfrac{1}{2}\,w - cae) \qquad (-\text{when } cae > \tfrac{1}{2}\,w),$$
$$\tfrac{1}{12} L\,(2\,h_s' + h_s)\,(\tfrac{1}{2}\,w' - cae') \qquad (-\text{when } cae' > \tfrac{1}{2}\,w').$$

31. Applicability of Rule 2. — This rule will give the volume, if there are no overhanging cliffs, not only of the railroad solid, but also of any solid satisfying the conditions of Art. 9, provided that one face lies in the reference plane.

If there are overhanging cliffs, use Rule 1, which differs from Rule 2 in that it takes into consideration the sign of the term when the forward height is to the left of the back height.

Rule 2 is applicable when C_1P, C_1Q, $C_1'P'$ and $C_1'Q'$ (Fig. 20) are four unequal distances; the slopes of the surfaces $BQQ'B'$ and $APP'A'$ may be unequal, and, moreover, these surfaces may be warped, so that the slopes of QB and $Q'B'$, as well as those of PA and $P'A'$, may be unequal. The straight line CC' does not necessarily lie in the surface, and hence it may not be a longitudinal edge.

32. Example. — Assuming that the length of the solid shown in Fig. 20 is 40 feet, the width of the road-bed PQ = 14 feet and $P'Q'$ = 20 feet, let the notes be,

$$\begin{array}{lcccccc}
 & A' & D' & C' & \multicolumn{2}{c}{B'} & \\
A'D'C'B' & \frac{25}{+15} & \frac{18}{+20} & \frac{0}{+18} & \multicolumn{2}{c}{\frac{20}{+10}} & (\tfrac{1}{2}\,w' = 10) \\
 & | \quad / & | & | & / & \backslash & \\
ADCEB & \frac{17}{+10} & \frac{6}{+16} & \frac{0}{+17} & \frac{5}{+18} & \frac{22}{+15} & (\tfrac{1}{2}\,w = 7) \\
 & A & D & C & E & B &
\end{array}$$

Then the terms corresponding to any edge other than an extreme edge, say DD', are found as follows (note that L = 40):

Place a pencil-point on the edge DD' (in the notes); then, starting with the end D, twice the height at that end + the height at the other end = $2 \times 16 + 20 = 52$; the adjacent edges are

AD' and CC', so that bfe is the horizontal distance between A and $C = 17 - 0 = 17$; hence the term is

$$\tfrac{1}{12} \times 40 \times 52 \times 17.$$

At the end D', twice the height at that end + the height at the other end $= 2 \times 20 + 16 = 56$; bfe' is the horizontal distance between D' and $C' = 18 - 0 = 18$; hence the term is

$$\tfrac{1}{12} \times 40 \times 56 \times 18.$$

The terms corresponding to an extreme edge, say BB', are found as follows: Starting at the end B, twice the height at that end + the height at the other end $= 2 \times 15 + 10 = 40$; the adjacent edge is EB', so that cae is the horizontal distance from C to $E = 5$, and $\frac{1}{2}w = 7$, and hence $\frac{1}{2}w - cae = 7 - 5 = 2$; therefore the term is

$$\tfrac{1}{12} \times 40 \times 40 \times 2.$$

At the end B', twice the height at that end + the height at the other end $= 2 \times 10 + 15 = 35$; cae' is the horizontal distance from C' to $B' = 20$, and $\frac{1}{2}w' = 10$, and hence $\frac{1}{2}w' - cae' = 10 - 20 = -10$; therefore the term is

$$\tfrac{1}{12} \times 40 \times 35 \times (-10).$$

The terms corresponding to each edge, with the resulting partial volumes, after dividing by 27 to reduce to cubic yards, are as follows:

Edge.	Terms.	+Vols.	−Vols.
AD'	$\frac{1}{324} \times 40 \times 40 \times 11$*	$54\frac{26}{81}$	
	" $\times 50 \times 7$†	$43\frac{17}{81}$	
DD'	" $\times 52 \times 17$	$109\frac{11}{81}$	
	" $\times 56 \times 18$	$124\frac{36}{81}$	
CC'	" $\times 52 \times 11$	$70\frac{50}{81}$	
	" $\times 53 \times 38$	$248\frac{52}{81}$	
EB'	" $\times 46 \times 22$	$124\frac{76}{81}$	
	" $\times 38 \times 20$	$93\frac{67}{81}$	
AA'	" $\times 35 \times (-10)$‡		$43\frac{17}{81}$
	" $\times 40 \times (-8)$‡		$39\frac{41}{81}$
BB'	" $\times 40 \times (+2)$§	$9\frac{71}{81}$	
	" $\times 35 \times (-10)$§		$43\frac{17}{81}$
	Totals =	$879\frac{1}{81}$	$125\frac{75}{81}$
		$125\frac{75}{81}$	
	Volume =	$753\frac{7}{81}$ cu. yds.	

* Between A and D since AA' is the back edge.

† Between A' and D' since DD' is the forward edge.

‡ AD' is the adjacent edge; $\frac{1}{2}w = 7$, $\frac{1}{2}w' = 10$.

§ EB' is the adjacent edge; $\frac{1}{2}w = 7$, $\frac{1}{2}w' = 10$.

THE RAILROAD SOLID WITH ONLY TRIANGULAR SURFACES AND PLANE SIDE SLOPES.

33. Mathematical Foundation of Rule 3. — Let Fig. 22 be the horizontal projection of the surface to the left of the center line C_1C_1' in Fig. 23, AA' being the extreme edge, and let the heights at the several vertices be represented by the correspond-

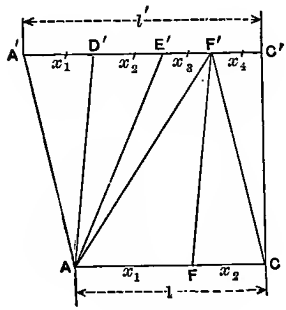

Fig. 22.

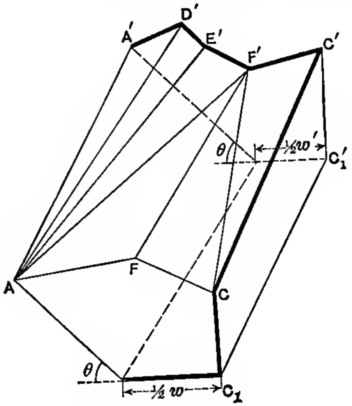

Fig. 23.

ing small letters. Then the volume below any one of the triangular surfaces is given by Eq. (6), Art. 15.

(1) *Consider any height except the extreme heights* (at A and A'). — The height f' at F' is common to the four truncated prisms $E'F'A$, $AF'F$, $FF'C$ and $CF'C'$; and the terms, in the expression for the volume, that contain f' are, by Eq. (6),

$$\tfrac{1}{3} f' \cdot \tfrac{1}{2} L\,(x_3' + x_1 + x_2 + x_4') = \tfrac{1}{6} Lf'\,[(x_3' + x_4') + (x_1 + x_2)],$$

which are given by the rule: — *For each height, except the extreme heights: Multiply the height by the horizontal distance between the adjacent heights* (not edges) *in its own end, and by the greatest horizontal distance at the other end between the longitudinal edges (if more than one) passing through the height; and multiply the sum of these products by the length of the solid* ÷ 6.

If we consider the height f at F, the corresponding terms are

$$\tfrac{1}{3} f \cdot \tfrac{1}{2} L\,(x_1 + x_2) = \tfrac{1}{6} Lf\,(x_1 + x_2),$$

which would be given by the same rule.

(2) *Consider the extreme heights.* — The height a at A is common to the truncated prisms $A'AD'$, $D'AE'$, $E'AF'$ and $F'AF$, and the corresponding terms in the expression for the volume are

$$\tfrac{1}{3} a \cdot \tfrac{1}{2} L\,(x_1 + x_1' + x_2' + x_3') = \tfrac{1}{6} La\,[x_1 + (x_1' + x_2' + x_3')].$$

The height a' at A' is in one prism $A'AD'$, and the corresponding term is

$$\tfrac{1}{3} a' \cdot \tfrac{1}{2} Lx_1' = \tfrac{1}{6} La'x_1'.$$

The volume under the plane side slope, which must be subtracted from the volume below the surface $ACC'A'$, is found by Eq. (10), where $h = a$, $h' = a'$, $m = l - \tfrac{1}{2} w$, and $m' = l' - \tfrac{1}{2} w'$. This volume is

$$\tfrac{1}{6} L\,[a(l - \tfrac{1}{2} w) + (a + a')\,(l' - \tfrac{1}{2} w')],$$

or $$\tfrac{1}{6} La\,[(l - \tfrac{1}{2} w) + (l' - \tfrac{1}{2} w')] + \tfrac{1}{6} La'\,(l' - \tfrac{1}{2} w').$$

Combining these results, the terms corresponding to a and a' are

$$\tfrac{1}{6} La\,[\tfrac{1}{2} w - (l - x_1) + \tfrac{1}{2} w' - (l' - x_1' - x_2' - x_3')]$$

and $$\tfrac{1}{6} La'\,[\tfrac{1}{2} w' - (l' - x_1')],$$

or $$\tfrac{1}{6} La\,(\tfrac{1}{2} w - CF + \tfrac{1}{2} w' - C'F') \text{ and } \tfrac{1}{6} La'\,(\tfrac{1}{2} w' - C'D'),$$

which are given by the rule: — *For each extreme height: Multiply the height by one half the width of the road-bed at its end — the distance out from the center to the adjacent height at the same end; and, if another edge passes through the height, multiply the height by one half the width of the road-bed at the other end — the distance out, at this other end, from the center to the edge through the height and farthest from the extreme edge; and multiply the sum of these products by the length of the solid* ÷ 6.

(3) To adapt the formulas to the slide rule, the height is multiplied by 2 and the result by $\tfrac{1}{12} L$ instead of $\tfrac{1}{6} L$, thus leading to Rule 3.

34. The Railroad Solid with only Triangular Surfaces and Plane Side Slopes. Rule 3. — (*a*) *For each height, except the extreme side heights: Multiply twice the height by the horizontal distance between the adjacent heights in its own end, and by the greatest horizontal distance at the other end between the longitudinal edges* (*if more than one*) *passing through the height; and multiply the sum of these products by the length of the solid* ÷ 12.

(*b*) *For each extreme height: Multiply twice the height by one half the width of the road-bed at its end* * — *the distance out* † *from the center to the adjacent height at the same end, and (if another edge passes through the height) by one half the width of the road-bed at the other end* ‡ — *the distance out,*† *at this other end, from the center to the edge through the height and farthest from the extreme edge; and multiply the sum of these products by the length of the solid* ÷ 12.

(*c*) *Add algebraically the results found by* (*a*) *and* (*b*).

Using the notation given in Appendix III, Rule 3 may be symbolized as follows:

35. Rule 3. The Railroad Solid with Triangular Surfaces and Plane Side Slopes.

For each height, except the extreme heights,

$$\frac{1}{12} L\,(2\,h)\,(dah + dd').$$

For each extreme height,

$$\frac{1}{12} L\,(2\,h_s)\,[(\tfrac{1}{2}\,w - cah) + (\tfrac{1}{2}\,w' - cfe')\S].$$

36. Rule 3 is Applicable when the distances C_1P, C_1Q, $C_1'P'$ and $C_1'Q'$ (Fig. 20) are four unequal quantities; the slopes of the plane surfaces $BQQ'B'$ and $APP'A'$ may be unequal; and the line CC' may or may not be a longitudinal surface edge.

37. Example. — In Figs. 22 and 23, let the length be 40 feet, the width of the road-bed at each end 20 feet, and the notes as follows:

$$\begin{array}{lccccc} & A' & D' & E' & F' & C' \\ A'D'E'F'C' & \dfrac{30}{+20} & \dfrac{16}{+28} & \dfrac{8}{+24} & \dfrac{4}{+16} & \dfrac{0}{+20} \\ & & & & & \\ AFC & & \multicolumn{2}{c}{\dfrac{18}{+8}} & \dfrac{12}{+24} & \dfrac{0}{+16} \\ & & \multicolumn{2}{c}{A} & F & C \end{array}$$

* On that side of the center line C_1C_1'.

† This distance out is negative if it and the extreme height are on opposite sides of the center of the road-bed.

‡ On that side of the center line C_1C_1', but at the other end.

§ This parenthesis is omitted when no edge other than the extreme edge passes through h_s.

Then the expressions corresponding to each height, with the resulting volumes, after dividing by 27 to reduce to cubic yards, are:

Point.	Terms.	+ Vols.	− Vols.
F	$\frac{1}{324} \times 40 \times 48 \times (18+0)$	$106\frac{54}{81}$	
C	" $\times 32 \times (12+4)$	$63\frac{17}{81}$	
C'	" $\times 40 \times (4+0)$	$19\frac{61}{81}$	
F'	" $\times 32 \times (8+18)$	$102\frac{58}{81}$	
E'	" $\times 48 \times (12+0)$	$71\frac{9}{81}$	
D'	" $\times 56 \times (22+0)$	$152\frac{8}{81}$	
A	" $\times 16 \times (10^* - 12 + 10^\dagger - 4)$	$7\frac{73}{81}$	
A'	" $\times 40 \times (10^\dagger - 16)$		$29\frac{51}{81}$
	Totals =	$523\frac{37}{81}$	$29\frac{51}{81}$
		$29\frac{51}{81}$	
	Volume =	$493\frac{67}{81}$ cu. yds.	

THREE-LEVEL SECTIONS WITH TWO WARPED SURFACES, THE WIDTH OF THE ROAD-BED BEING CONSTANT.

38. Three-Level Sections. — A three-level section is one in which the cross-section of the surface is determined by three points, — A and B, where the side slopes intersect the surface, and C vertically above the center of the road-bed.

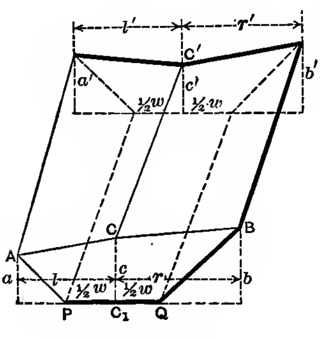

Fig. 24.

39. Area of a Three-Level Section. — In Fig. 25, the cross-section $PACBQ$ is composed of the four triangles AC_1P, BC_1Q, AC_1C and BC_1C, and its area is

$$\tfrac{1}{2}\, a \frac{w}{2} + \tfrac{1}{2}\, b\, \frac{w}{2} + \tfrac{1}{2}\, c\, l + \tfrac{1}{2}\, c\, r,$$

or

$$\tfrac{1}{2}\,(a + b)\,\frac{w}{2} + \tfrac{1}{2}\, c\,(l + r).$$

Similarly, the area of the other end (Fig. 24) is

$$\tfrac{1}{2}\,(a' + b')\,\frac{w}{2} + \tfrac{1}{2}\, c'(l' + r'),$$

* One half width of the road-bed in the end AFC.

† One half width of the road-bed in the end $A'D'E'F'C'$.

and the area of the mid-section (Fig. 26) is

$$\tfrac{1}{4}(a + a' + b + b')\frac{w}{2} + \tfrac{1}{8}(c + c')(l + l' + r + r').$$

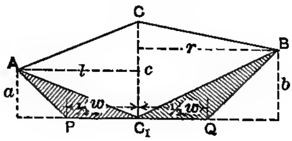

Fig. 25.

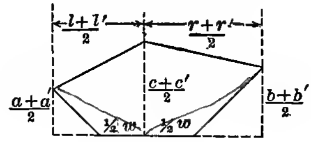

Fig. 26.

40. Formula for the Volume. — Substituting these expressions for the areas in Eq. (1), we have, for the volume,

$$P = \tfrac{1}{6} L \left[\tfrac{1}{2}(a + b)\frac{w}{2} + \tfrac{1}{2} c (l + r) + \tfrac{1}{2}(a' + b')\frac{w}{2} + \tfrac{1}{2} c' (l' + r')\right.$$
$$\left. + (a + a' + b + b')\frac{w}{2} + \tfrac{1}{2}(c + c')(l + l' + r + r')\right]$$
$$= \tfrac{1}{4} L (a + b + a' + b')\frac{w}{2} + \tfrac{1}{12} L (2c + c')(l + r)$$
$$+ \tfrac{1}{12} L (2c' + c)(l' + r'),$$

which will be given by Rule 4.

41. Three-Level Sections with Two Warped Surfaces, the Width of the Road-Bed being Constant. Rule 4. — (*a*) *Multiply the sum of the four side heights by* $\frac{1}{2} w$ *and the product by* $\frac{1}{4} L$.

(*b*) *At each end, to twice the center height at that end add the center height at the other end, and multiply the sum by the sum of the distances out to the side heights at the first end, and multiply the product by* $\frac{1}{12} L$.

(*c*) *Add the results of* (*a*) *and* (*b*).

This rule may be symbolized as follows:

42. Rule 4. Three-Level Sections with Two Warped Surfaces.

$$\tfrac{1}{4} L (a + b + a' + b')\frac{w}{2},$$

$$\tfrac{1}{12} L (2c + c')(l + r),$$

$$\tfrac{1}{12} L (2c' + c)(l' + r').$$

43. General Applicability of Rule 4. — Rule 4 is applicable to any railroad prismoid (with a constant width of road-bed) in which there are three, and only three, longitudinal edges, *the*

middle one being in the same vertical plane with the center line of the road-bed, and the other two being the intersections of the surface either with the side slopes or with the road-bed, as the case may be; * thus, it is applicable to the solids † shown in

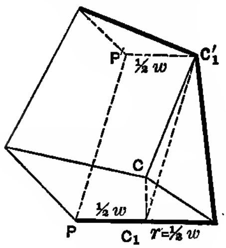

Fig. 27.

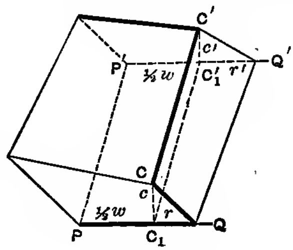

Fig. 28.

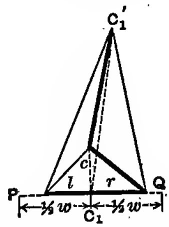

Fig. 29.

Figs. 27, 28, 29, 30 and 31, as well as to others, but it does not apply to Fig. 32, since this solid does not have a longitudinal edge in the same vertical plane with the center line C_1C_1'.

The rule is applicable when the slopes of the surfaces $BQQ'B'$

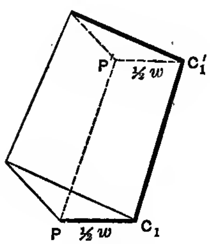

Fig. 30.

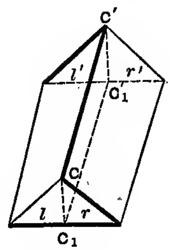

Fig. 31.

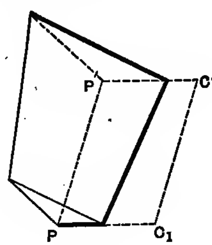

Fig. 32.

and $APP'A'$ (Fig. 20) are equal or unequal; and moreover, these surfaces may be warped, so that the slopes of QB and $Q'B'$, as well as those of PA and $P'A'$, may be unequal.

If the width of the road-bed differs at the two ends, use Rule 2.

* If the surface intersects the road-bed, the corresponding side heights (a and a' or b and b') are zero, and the corresponding values of l and l' or of r and r' are the horizontal distances from the center to this intersection; the distance from the center to the edge of the road-bed on that side of the center line may be different at the two ends.

† In Fig. 30, two longitudinal edges may be considered as coinciding with C_1C_1'.

44. Example. — The width of the road-bed being 14 feet, and the distance between stations 100 feet, let the notes be:

	Station.	Cross-Sections.		
Sec 3.	83.	$\frac{25}{+12}$	$\frac{0}{+15}$	$\frac{34}{+18}$
Sec. 2.	82.	$\frac{31}{+16}$	$\frac{0}{+20}$	$\frac{28}{+14}$
Sec. 1.	81 + 40.	$\frac{37}{+20}$	$\frac{0}{+30}$	$\frac{40}{+22}$

The distance between Sec. 1 and Sec. 2 is 60 (= 100 − 40) feet, and that between Sec. 2 and Sec. 3 is 100 feet. Hence the terms in the volume between Sec. 1 and Sec. 2 are, in cubic yards,

$$
\begin{aligned}
\tfrac{1}{108} \times 60 \times 72 \times 7 &= 280 \quad \text{cu. yds.}\\
\tfrac{1}{324} \times 60 \times 80 \times 77 &= 1140\tfrac{20}{27} \quad \text{"}\\
\tfrac{1}{324} \times 60 \times 70 \times 59 &= 764\tfrac{22}{27} \quad \text{"}\\
\text{Total} &= 2185\tfrac{5}{9} \quad \text{cu. yds.}
\end{aligned}
$$

The volume between Sec. 2 and Sec. 3 is, in cubic yards,

$$
\begin{aligned}
\tfrac{1}{108} \times 100 \times 60 \times 7 &= 388\tfrac{72}{81} \quad \text{cu. yds.}\\
\tfrac{1}{324} \times 100 \times 55 \times 59 &= 1001\tfrac{44}{81} \quad \text{"}\\
\tfrac{1}{324} \times 100 \times 50 \times 59 &= 910\tfrac{40}{81} \quad \text{"}\\
\text{Total} &= 2300\tfrac{25}{27} \quad \text{cu. yds.}
\end{aligned}
$$

THREE-LEVEL SECTIONS WITH TWO WARPED SURFACES, THE WIDTH OF THE ROAD-BED BEING CONSTANT, AND WITH PLANE SIDE SLOPES OF EQUAL INCLINATION.

45. The Grade Prism. — If the side slopes $BQQ'B'$ and $APP'A'$ (Fig. 33) are prolonged, they will intersect in the line VV', parallel to C_1C_1' and in the vertical plane through C_1C_1'.

Let $\tan AVC = \tan BVC = \tan A'V'C' = \tan B'V'C' = s$, and let $C_1V = C_1'V' = c_0$.

$$\therefore c_0 = \frac{w}{2s}.$$

The area of the triangle PQV is $\frac{1}{2}c_0 w = \frac{w^2}{4\,s}$; and hence the volume of the triangular prism PQV-$P'Q'V'$, called the **grade prism,** is

$$G = \frac{Lw^2}{4\,s}$$

46. Formula for the Volume. — The end $ACBV$ is formed by the two triangles AVC and BVC, and its area is

$$\tfrac{1}{2}(c + c_0)(l + r).$$

The area of the end $A'C'B'V'$ is

$$\tfrac{1}{2}(c' + c_0)(l' + r').$$

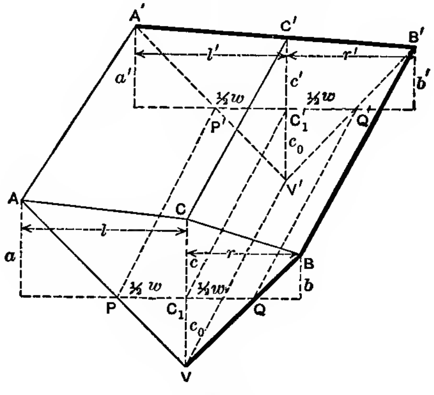

Fig. 33.

The area of the mid-section is

$$\tfrac{1}{2}\left(\frac{c + c'}{2} + c_0\right)\left(\frac{l + l'}{2} + \frac{r + r'}{2}\right).$$

Hence, by Eq. (1), the volume $ACBV$-$A'C'B'V'$ is

$$\tfrac{1}{6}L\left[\tfrac{1}{2}(c + c_0)(l + r) + \tfrac{1}{2}(c' + c_0)(l' + r') + \tfrac{1}{2}(c + c' + 2\,c_0)(l + r + l' + r')\right],$$

or

$$\tfrac{1}{12}L(2\,c + c' + 3\,c_0)(l + r) + \tfrac{1}{12}L(2\,c' + c + 3\,c_0)(l' + r').$$

Hence the volume above the road-bed $PQQ'P'$ is

$$P = \tfrac{1}{12} L\,(2\,c + c' + 3\,c_0)\,(l + r) + \tfrac{1}{12} L\,(2\,c' + c + 3\,c_0)\,(l' + r') - \frac{Lw^2}{4\,s}\,.$$

The corresponding rule may be stated as follows:

47. Rule 5. Three-Level Sections with Two Warped Surfaces, using the Grade Prism.

$$\tfrac{1}{12} L\,(2\,c + c' + 3\,c_0)\,(l + r),$$
$$\tfrac{1}{12} L\,(2\,c' + c + 3\,c_0)\,(l' + r'),$$
$$-\frac{Lw^2}{4\,s}\,.$$

48. The Application of Rule 5 is limited to solids in which the surface intersects both side slopes, the width of the road-bed being constant, the plane side slopes making the same angle with a horizontal plane and intersecting each other in the same vertical plane with the center line of the road-bed. The straight line CC' must be a longitudinal surface edge.

49. Example. — The width of the road-bed being 14 feet and the side slope ratio s being $1\frac{1}{2}$ to 1 (horizontal ÷ vertical), so that $\tan AVC = \frac{3}{2}$, let the notes be,

	Station.	Cross-Sections.		
Sec. 2.	82.	$\frac{31}{+16}$	$\frac{0}{+20}$	$\frac{28}{+14}$
Sec. 1.	81 + 40.	$\frac{37}{+20}$	$\frac{0}{+30}$	$\frac{40}{+22}$

In this case $c_0 = w \div 2\,s = \frac{14}{3}$, and the volume between Sec. 1 and Sec. 2 ($L = 60$) is, in cubic yards,

$$\begin{aligned}
\tfrac{1}{324} \times 60 \times 94 \times 77 &= 1340\tfrac{10}{27} \\
\text{“} \quad\quad \times 84 \times 59 &= 917\tfrac{21}{27} \\
&\overline{2258\tfrac{4}{27}} \\
\tfrac{1}{27}\,\frac{Lw^2}{4\,s} = \tfrac{60}{108} \times 14^2 \times \tfrac{2}{3} &= 72\tfrac{16}{27} \\
\therefore \text{Volume} &= \overline{2185\tfrac{5}{9}} \text{ cu. yds.}
\end{aligned}$$

THREE-LEVEL SECTIONS WITH PLANE SURFACES, THE WIDTH OF THE ROAD-BED BEING CONSTANT, AND WITH PLANE SIDE SLOPES OF EQUAL OR OF UNEQUAL INCLINATION.

50. The Plane Surfaces. — In Figs. 34 and 35, the surface is composed of four triangles, formed by the three edges AA', CC' and BB' together with two **diagonal edges,** each of which is drawn from one of the center heights to one of the side heights (AC' and CB' in Fig. 34, and CA' and CB' in Fig. 35).

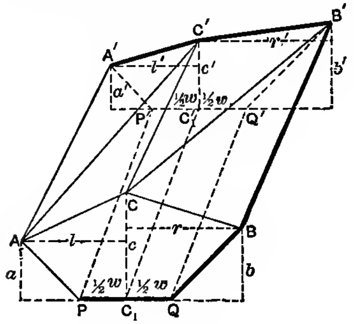

Fig. 34.

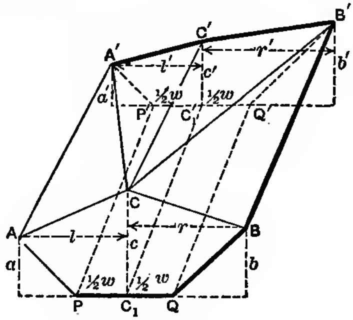

Fig. 35.

51. Derivation of the Formula. — The two possible cases are shown in Figs. 34 and 35.

(1) Fig. 34. — Applying Rule 3, we have

$$\begin{aligned} P &= \tfrac{1}{12} L\,[2\,c\,(l + r + r') + 2\,c'\,(l' + r' + l) + 2\,a\,(\tfrac{1}{2}\,w + \tfrac{1}{2}\,w) \\ &\qquad + 2\,a'\,(\tfrac{1}{2}\,w) + 2\,b\,(\tfrac{1}{2}\,w) + 2\,b'\,(\tfrac{1}{2}\,w + \tfrac{1}{2}\,w)] \\ &= \tfrac{1}{12} L\,[(a + a' + b + b') + (a + b')]\,w \\ &\qquad + \tfrac{1}{12} L\,(2\,c)\,[(l + r) + r'] + \tfrac{1}{12} L\,(2\,c')\,[(l' + r') + l]. \end{aligned}$$

(2) Fig. 35. — Applying Rule 3, we have

$$\begin{aligned} P &= \tfrac{1}{12} L\,[2\,c\,(l + r + l' + r') + 2\,c'\,(l' + r') + 2\,a\,(\tfrac{1}{2}\,w) \\ &\qquad + 2\,a'\,(\tfrac{1}{2}\,w + \tfrac{1}{2}\,w) + 2\,b\,(\tfrac{1}{2}\,w) + 2\,b'\,(\tfrac{1}{2}\,w + \tfrac{1}{2}\,w)] \\ &= \tfrac{1}{12} L\,[(a + b + a' + b') + (a' + b')]\,w \\ &\qquad + \tfrac{1}{12} L\,(2\,c)\,[(l + r) + (l' + r')] + \tfrac{1}{12} L\,(2\,c')(l' + r'). \end{aligned}$$

In each term of both of these formulas there are three factors, as follows: (*a*) $\tfrac{1}{12} L$, w and $a + b + a' + b'$ + the sum of the side heights at the ends of the diagonals ($= \Sigma\, dsh$);

(*b*) $\frac{1}{12}L$, $2c$ and $l + r +$ the distances out, at the other end, to the ends of the diagonals ($= \Sigma\, ddo'$);

(*c*) $\frac{1}{12}L$, $2c'$ and $l' + r' +$ the distances out, at the other end, to the ends of the diagonals ($= \Sigma\, ddo$).

Using the notation in Appendix III, the terms may be symbolized as follows:

52. Rule 6. Three-Level Sections with Plane Surfaces.

$$(a)\quad \tfrac{1}{12}Lw\,(a + b + a' + b' + \Sigma\, dsh),$$
$$(b)\quad \tfrac{1}{12}L\,(2\,c)\ (l + r + \Sigma\, ddo'),$$
$$(c)\quad \tfrac{1}{12}L\,(2\,c')\ (l' + r' + \Sigma\, ddo).$$

53. Example. — Let the notes for a prismoid 60 feet long, $w = 14$, be

$$\text{Sec. 2.}\quad \frac{22}{+10}\quad \frac{0}{+12}\quad \frac{28}{+14}$$
$$\text{Sec. 1.}\quad \frac{25}{+12}\quad \frac{0}{+15}\quad \frac{31}{+16}$$

The side heights at the ends of the diagonals are 12 and 16; the distances out to the ends of the diagonals are, in Sec. 1, 25 and 31, and, in Sec. 2, zero. Hence the volume is, in cubic yards,

$$\begin{array}{llrl}
(a) & \frac{1}{324}\times 60\times 14 \times (52 + 28) & = & 207\tfrac{11}{27}\\
(b) & \quad ''\quad \times 30 \times (56 + \ \ 0) & = & 311\tfrac{3}{27}\\
(c) & \quad ''\quad \times 24 \times (50 + 56) & = & 471\tfrac{3}{27}\\
 & \qquad\qquad \text{Volume} & = & 989\tfrac{17}{27}\ \text{cu. yds.}
\end{array}$$

SPECIAL CASES.

54. Five-Level Sections, Two Levels above the Edges of the Road-Bed, with four warped surfaces, the width of the road-bed being constant, the side slopes either plane or warped. When the solid is in the form in Fig. 36, the volume is, by Rule 2,

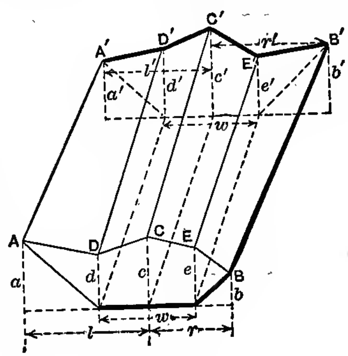

Fig. 36.

$$P = \tfrac{1}{12}L\,[(2\,d + d')\,l + (2\,d' + d)\,l' + (2\,c + c')\,w + (2\,c' + c)\,w + (2\,e + e')\,r + (2\,e' + e)\,r'];$$

or the sum of the terms:

$$\textbf{Rule 7:}\quad \begin{cases} \frac{1}{4} L\,(c + c')\,w, \\ \frac{1}{12} L\,(2d + d')\,l, \\ \frac{1}{12} L\,(2\,d' + d)\,l', \\ \frac{1}{12} L\,(2\,e + e')\,r, \\ \frac{1}{12} L\,(2\,e' + e)\,r'. \end{cases}$$

55. Example. — Let the notes for a prismoid 80 feet long, $w = 14$ feet, be

$$\begin{array}{lccccc} \text{Sec. 2.} & \frac{100}{+62} & \frac{7}{+50} & \frac{0}{+40} & \frac{7}{+30} & \frac{37}{+20} \\ & | & | & | & | & | \\ \text{Sec. 1.} & \frac{130}{+82} & \frac{7}{+70} & \frac{0}{+60} & \frac{7}{+50} & \frac{70}{+42} \end{array}$$

Then the volume, in cubic yards, according to Rule 7, is:

$$\begin{array}{lllcl} \frac{1}{108} \times 80 & \times 100 & \times 14 & = & 1037\frac{3}{81} \\ \frac{1}{324} \times 80 & \times 190 & \times 130 & = & 6098\frac{62}{81} \\ \quad " & \times 170 & \times 100 & = & 4197\frac{43}{81} \\ \quad " & \times 130 & \times 70 & = & 2246\frac{74}{81} \\ \quad " & \times 110 & \times 37 & = & 1004\frac{76}{81} \\ & & \text{Volume} & = & 14585\frac{5}{27} \text{ cu. yds.} \end{array}$$

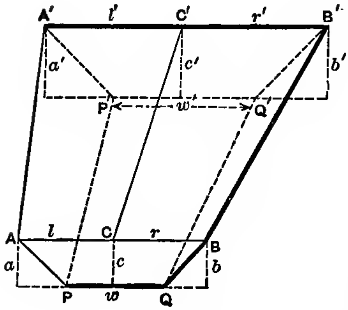

Fig. 37.

56. Level Sections. — By Rule 2, the volume is

$$\tfrac{1}{12} L\,[(2\,c + c')\,(l + r) + (2\,c' + c)\,(l' + r') + (2\,a + a')\,\tfrac{1}{2}\,w + (2\,a' + a)\,\tfrac{1}{2}\,w' + (2\,b + b')\,\tfrac{1}{2}\,w + (2\,b' + b)\,\tfrac{1}{2}\,w'];$$

or, since $a = b = c$, and $a' = b' = c'$, this is equivalent to the sum of the two expressions,

$$\textbf{Rule 8 (a):}\quad \begin{cases} \frac{1}{12} L\,(2\,c + c')\,(l + r + w), \\ \frac{1}{12} L\,(2\,c' + c)\,(l' + r' + w'). \end{cases}$$

Rule 8 (a) is applicable when the surfaces $BQQ'B'$ and $APP'A'$ are either plane or warped, and when their slopes are equal or unequal.

If the surfaces $BQQ'B'$ and $APP'A'$ are plane, and if they make the same angle with the horizontal plane, we have,

$$l = r = \tfrac{1}{2} w + cs, \quad \text{and} \quad l' = r' = \tfrac{1}{2} w' + c's,$$

and the expressions for the volume become

$$\textbf{Rule 8 (b):} \quad \begin{cases} \frac{1}{12} L\,(2\,c + c')\,2\,(w + cs), \\ \frac{1}{12} L\,(2\,c' + c)\,2\,(w' + c's). \end{cases}$$

If, in Fig. 33, the surface $ACBB'C'A'$ were in a horizontal plane, we should have

$$l + r = 2\,(c + c_0)\,s \quad \text{and} \quad l' + r' = 2\,(c' + c_0)\,s.$$

Substituting these expressions in the formula for P in Art. 46, we have

$$P = \tfrac{1}{12} L\,(2\,c + c' + 3\,c_0)\,(c + c_0) \cdot 2\,s$$
$$+ \tfrac{1}{12} L\,(2\,c' + c + 3\,c_0)\,(c' + c_0) \cdot 2\,s - \frac{L\,w^2}{4\,s},$$

or

$$P = \tfrac{1}{12} L\,[(c + c_0)^2 + (c' + c_0)^2 + (c + c_0)\,(c' + c_0)] \cdot 4\,s - \frac{L\,w^2}{4\,s}.$$

This formula is applicable when the width of the road-bed is constant and the plane side slopes are equally inclined to the horizontal plane.

57. Ditches. — Rules 8 (a) and 8 (b) may be used to find the volumes of the ditches.

58. When the Side Slope is Afterwards Flattened. — In Fig. 38, let $BQQ'B'$ be the original plane side slope, and $B_1QQ'B_1'$ the plane side slope after the volume BQB_1-$B'Q'B_1'$ has been removed; and let the old side slope ratio = tan $\theta = s$, and the new side slope ratio = tan $\theta_1 = s_1$.

Then, by Rule 1, the volume BQB_1-$B'Q'B_1'$ is

$$\tfrac{1}{12} L\,[(2\,b + b')\,b_1 s_1 + (2\,b' + b)\,b_1' s_1 + (2\,b_1 + b_1')\,(-bs)$$
$$+ (2\,b_1' + b_1)\,(-\,b's)],$$

which reduces to the sum of the two terms,

$$\textbf{Rule 9:} \quad \begin{cases} \frac{1}{12} L\,(2\,b_1 + b_1')\,b\,(s_1 - s), \\ \frac{1}{12} L\,(2\,b_1' + b_1)\,b'\,(s_1 - s). \end{cases}$$

59. Sidings. — In Fig. 39, QQ' is the edge of the original road-bed, and $QQ'Q_1'Q_1$ is the area added to the road-bed to allow for the siding, $BQQ'B'$ and $B_1Q_1Q_1'B_1'$ being plane surfaces.

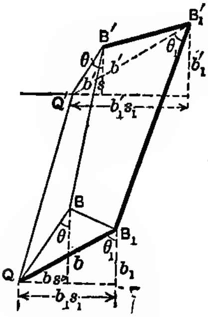

Fig. 38.

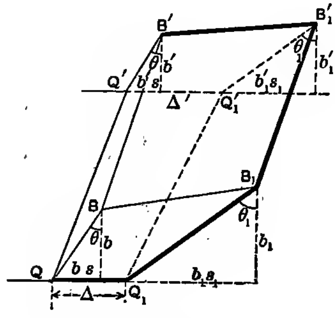

Fig. 39.

(1) *The General Case*, shown in Fig. 39. — By Rule 1, the volume $B_1Q_1QB\text{-}B_1'Q_1'Q'B$ is

$$\tfrac{1}{12} L\,[(2\,b + b')\,(\Delta + b_1 s_1) + (2\,b' + b)\,(\Delta' + b_1' s_1) + (2\,b_1 + b_1')\,(\Delta - bs) + (2\,b_1' + b_1)\,(\Delta' - b's)],$$

which reduces to the sum of the four terms,

Rule 10 (a):

$$\begin{cases} \tfrac{1}{12} L\,[2\,(b + b_1) + (b' + b_1')]\,\Delta, \\ \tfrac{1}{12} L\,[2\,(b' + b_1') + (b + b_1)]\,\Delta', \\ \tfrac{1}{12} L\,(2\,b_1 + b_1')\,b\,(s_1 - s), \\ \tfrac{1}{12} L\,(2\,b_1' + b_1)\,b'\,(s_1 - s). \end{cases}$$

(2) *When* $s_1 = s$. — Rule 10 (a) becomes

Rule 10 (b):

$$\begin{cases} \tfrac{1}{12} L\,[2\,(b + b_1) + (b' + b_1')]\,\Delta, \\ \tfrac{1}{12} L\,[2\,(b' + b_1') + (b + b_1)]\,\Delta'. \end{cases}$$

(3) *When* $s_1 = s$ *and* $\Delta' = \Delta$. — Rule 10 (a) becomes

Rule 10 (c): $\tfrac{1}{4} L\,(b + b_1 + b' + b_1')\,\Delta.$

(4) *When* $\Delta = \Delta' = 0$. — Rule 10 (a) becomes the same as Rule 9.

BORROW PITS AND GRADING.

60. Description. — In making embankments it is sometimes cheaper, when the excavated earth in the neighborhood of the embankment is insufficient, to take the additional quantity from a nearby field than to bring it from a more distant cut. Before the work is begun, the surface of the ground is divided into rectangles of the same size with convenient dimensions, and levels taken at the corners; after the earth is removed, levels are again taken at the same points; and from the data thus obtained, the excavated volume is computed. The same method may be used to determine the volume removed or added in grading.

There are two cases, — first, when the rectangles are subdivided into triangles so that the surface is considered as composed of plane triangles; and, second, when the surface is considered as formed by quadrilaterals, either plane or warped.

NOTE. — The volume between the original surface and a horizontal reference plane below the excavation may be determined, then the volume between the new surface and the reference plane, and the second volume subtracted from the first.

61. When the Surface is composed of Triangles. — Let the width of each horizontal rectangle be w and its length L, as in Fig. 40, and let the direction of the diagonal line that lies in the surface be noted. The volume is then considered as composed of truncated triangular prisms, all having the same basal area. Since, from Eq. (6), the volume of such a prism equals the area of the base multiplied by one third of the sum of the vertical edges, this constant basal area must be multiplied by one third of a given vertical edge, taken as many times as there are prisms to which it belongs. Thus, the height a at A is in one prism, and the corresponding term in the volume is $(\frac{1}{2}Lw)\cdot\frac{1}{3}a$; the height b at B is in two prisms, and the sum of the corresponding terms is $(\frac{1}{2}Lw)\cdot 2\,(\frac{1}{3}b)$; the height c at C is in three prisms, and the sum of the corresponding terms is $(\frac{1}{2}Lw)\cdot 3\,(\frac{1}{3}c)$; the height d at D is in eight prisms, the largest possible number, and the corresponding terms equal $(\frac{1}{2}Lw)\cdot 8\,(\frac{1}{3}d)$.

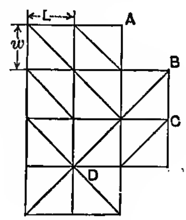

Fig. 40.

If subscripts are used to denote the number of prisms to which a height belongs, and Σ is read "the sum of," so that Σh_3 means "the sum of the vertical edges, each of which is common to three triangular prisms," the volume is the sum of eight terms, forming

$$\textbf{Rule 11 (a):} \begin{cases} \frac{1}{12} L\,(2\,w)\,[\Sigma h_1 + 2\,\Sigma h_2 + 3\,\Sigma h_3 + 4\,\Sigma h_4 \\ \qquad + 5\,\Sigma h_5 + 6\,\Sigma h_6 + 7\,\Sigma h_7 + 8\,\Sigma h_8]. \end{cases}$$

NOTE. — One or more of these eight terms may be zero.

62. When the Surface is composed of Quadrilaterals, Plane or Warped. — Let w and L be the common breadth and length of the rectangular bases. From Eq. (2), making $m' = m = w$, the volume in Fig. 4, with a rectangular base, is

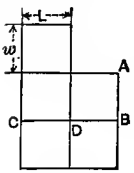

Fig. 41.

$$Lw \times \tfrac{1}{4}\,(h_1 + h_2 + h_1' + h_2'),$$

or

the area of the base $\times$ $\frac{1}{4}$ *the sum of the vertical edges.*

Hence, if subscripts are used to denote the number of prisms to which a height belongs, and Σ is read "the sum of," so that Σh_3 means "the sum of the vertical edges, each of which is common to three rectangular prisms," the volume is the sum of four terms, forming

$$\textbf{Rule 11 (b):}\quad \tfrac{1}{4} Lw\,[\,\Sigma h_1 + 2\,\Sigma h_2 + 3\,\Sigma h_3 + 4\,\Sigma h_4].$$

RAILROAD SIDE-HILL WORK.

63. The Preceding Methods are Sufficient for the computation of side-hill work, but two special rules may be formulated.

64. Triangular Ends with a Warped Surface, Fig. 42. — Let $BQQ'B'$ be the side slope, plane or warped, $BSS'B'$ the surface, and SS' its intersection with the road-bed. Then, by Rule 1, the volume of $BSQ\text{-}B'S'Q'$ is given by the sum of the two terms,

$$\textbf{Rule 12:} \begin{cases} \frac{1}{12} L\,(2\,h + h')\,m, \\ \frac{1}{12} L\,(2\,h' + h)\,m'. \end{cases}$$

65. Triangular Ends with Triangular Plane Surfaces, Figs. 43 and 44. — In Fig. 43, the volume is composed of the triangular

pyramid B-$B'Q'S'$ and the trapezoidal pyramid B-$S'Q'QS$, so that the volume is

$$\tfrac{1}{3} L \cdot \tfrac{1}{2} h'm' + \tfrac{1}{3} h \cdot L \frac{m + m'}{2},$$

or

$$\tfrac{1}{12} L \,[2\, mh + 2\, m' \,(h + h')].$$

In Fig. 44, the volume is composed of the triangular pyramid

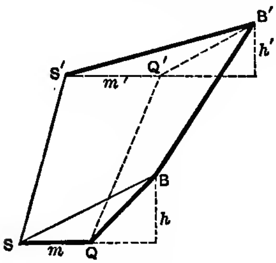

Fig. 42.

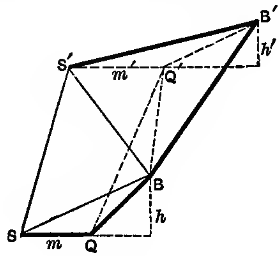

Fig. 43.

B'-BQS and the trapezoidal pyramid B'-$S'Q'QS$, so that the volume is

$$\tfrac{1}{3} L \cdot \tfrac{1}{2} hm + \tfrac{1}{3} h' \cdot L \frac{m + m'}{2},$$

or

$$\tfrac{1}{12} L \,[2\, m \,(h + h') + 2\, m'h'].$$

If the heights at the ends of the diagonal be dh and dh', the expression for the volume, in either case, consists of the two terms,*

Rule 13: $\begin{cases} \frac{1}{12} L\,(2\,m)\,(h + dh'), \\ \frac{1}{12} L\,(2\,m')\,(h' + dh). \end{cases}$

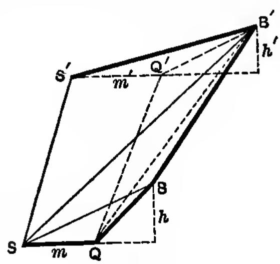

Fig. 44.

66. The Necessary Measurements.—When great accuracy is required in side-hill work, cross-sections should be taken (Fig. 45),

(1) where the surface cuts each edge of the embankment road-bed (C and E),

* One of the diagonal heights will be zero, and the other will be h or h', as the case may be.

(2) where the center line pierces the road-bed (D), and

(3) where the thorough cut begins with the full width of the road-bed (F); and

(4) the horizontal distance to the point B, where the cut runs out, should be measured.

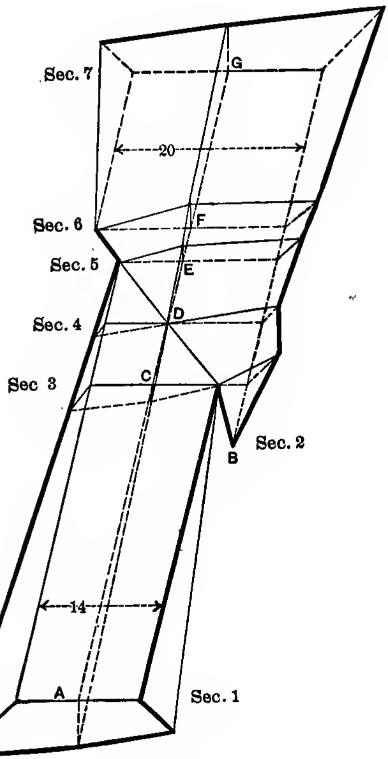

Fig. 45.

The preceding rules will serve for the determination of the volumes shown in Fig. 45.

67. Example. — The width of the road-bed being 20 feet in cut and 14 feet in fill, let the notes for Fig. 45 be:

	Station.*		Cross-Sections.		
Sec. 7.	22.	$\frac{13}{+2}$	$\frac{0}{+3}$	$\frac{16}{+4}$	
Sec. 6.	+75.	$\frac{10}{0}$	$\frac{0}{+1.2}$	$\frac{13}{+2}$	
Sec. 5.	+70.	$\frac{7}{0}$	$\frac{0}{+1}$	$\frac{12.1}{+1.4}$	
Sec. 4.	+60.	$\frac{8.5}{-1}$	$\frac{0}{0}$	$\frac{11.5}{+1}$	
Sec. 3.	+50.	$\frac{9.4}{-1.6}$	$\frac{0}{-1}$	$\frac{7}{0}$	$\frac{13}{+2}$
Sec. 2.	+40.				$\frac{10}{0}$
Sec. 1.	21.	$\frac{12.1}{-3.4}$	$\frac{0}{-2.8}$	$\frac{10}{-2}$	

where the distance out is in the numerator and the corresponding height in the denominator, a positive sign indicating a cut and a negative a fill, and the lines indicate the longitudinal edges of the solid.

Each of the volumes may be found either by Rule 2 or by Rule 4, dividing by 27 to reduce to cubic yards, as follows:

(1) Fill between Sec. 1 and Sec. 3. — Rule 4.

$$\tfrac{1}{108} \times 50 \times 7.0 \times 7 = 22\tfrac{222}{324}$$
$$\tfrac{1}{324} \times 50 \times 6.6 \times 22.1 = 22\tfrac{165}{324}$$
$$\tfrac{1}{324} \times 50 \times 4.8 \times 16.4 = 12\tfrac{48}{324}$$
$$\text{Volume} = 57\tfrac{111}{324} \text{ cu. yds.}$$

(2) Fill between Sec. 3 and Sec. 4. — Rule 4.

$$\tfrac{1}{108} \times 10 \times 2.6 \times 7 = 1\tfrac{222}{324}$$
$$\tfrac{1}{324} \times 10 \times 2.0 \times 16.4 = 1\tfrac{4}{324}$$
$$\tfrac{1}{324} \times 10 \times 1.0 \times 8.5 = \tfrac{85}{324}$$
$$\text{Volume} = 2\tfrac{311}{324} \text{ cu. yds.}$$

* Sec. 2 is 40 feet from Sec. 1; Sec. 3 is 50 feet from Sec. 1; and so on, Sec. 7 being 100 feet from Sec. 1.

(3) Fill between Sec. 4 and Sec. 5. — Rule 2, part (*b*).

$$\tfrac{1}{324} \times 10 \times 2 \times 7 = \tfrac{35}{81} \text{ cu. yds.}$$

(4) Cut between Sec. 2 and Sec. 3. — Rule 2, part (*b*).

$$\tfrac{1}{324} \times 10 \times 4 \times 3 = \tfrac{10}{27} \text{ cu. yds.}$$

(5) Cut between Sec. 3 and Sec. 4. — Rule 2, part (*b*).

$$\begin{aligned}
\tfrac{1}{324} \times 10 \times 5 \times 3 &= \tfrac{150}{324} \\
\tfrac{1}{324} \times 10 \times 4 \times 10 &= 1\tfrac{76}{324} \\
\text{Volume} &= 1\tfrac{226}{324} \text{ cu. yds.}
\end{aligned}$$

(6) Cut between Sec. 4 and Sec. 5. — Rule 4.

$$\begin{aligned}
\tfrac{1}{108} \times 10 \times 2.4 \times 10 &= 2\tfrac{72}{324} \\
\tfrac{1}{324} \times 10 \times 1.0 \times 11.5 &= \tfrac{115}{324} \\
\tfrac{1}{324} \times 10 \times 2.0 \times 19.1 &= 1\tfrac{58}{324} \\
\text{Volume} &= 3\tfrac{245}{324} \text{ cu. yds.}
\end{aligned}$$

(7) Cut between Sec. 5 and Sec. 6. — Rule 4.

$$\begin{aligned}
\tfrac{1}{108} \times 5 \times 3.4 \times 10 &= 1\tfrac{186}{324} \\
\tfrac{1}{324} \times 5 \times 3.2 \times 19.1 &= \tfrac{305.6}{324} \\
\tfrac{1}{324} \times 5 \times 3.4 \times 23 &= 1\tfrac{67}{324} \\
\text{Volume} &= 3\tfrac{391}{540} \text{ cu. yds.}
\end{aligned}$$

(8) Cut between Sec. 6 and Sec. 7. — Rule 4.

$$\begin{aligned}
\tfrac{1}{108} \times 25 \times 8 \times 10 &= 18\tfrac{56}{108} \\
\tfrac{1}{324} \times 25 \times 5.4 \times 23 &= 9\tfrac{63}{108} \\
\tfrac{1}{324} \times 25 \times 7.2 \times 29 &= 16\tfrac{12}{108} \\
\text{Volume} &= 44\tfrac{23}{108} \text{ cu. yds.}
\end{aligned}$$

CHAPTER II.

APPLICATION OF THE AVERAGE END AREA METHOD WHEN THE CROSS-SECTIONS ARE DETERMINED BY LEVELS.

68. Theory of the Average End Area Method. — In this method, an approximate value of the volume is found by multiplying the average of the two end areas by the length; thus, if A and A' are the areas of the two parallel ends, and L is the perpendicular distance between them, the volume by this method would be

$$E = L\left(\frac{A + A'}{2}\right),$$

or

$$E = \tfrac{1}{2}LA + \tfrac{1}{2}LA' \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

that is, the volume is considered as equivalent to two right prisms, each with the length $\frac{1}{2}L$, one having the cross-section A, and the other A'.

In this method, no attention is paid to the directions of the longitudinal edges, and the results obtained are only approximate. Nevertheless it is extensively used on account of the simplicity of the necessary computations, and because approximate values of the volumes are usually all that are required in railroad work.

In Chapter I, volumes have been determined by the prismoidal formula, using rules that may be easily applied. In Chapter II, some of these volumes will be determined by the average end area method; and in Chapter III, we shall find the correction to be applied to the average end area volume to obtain the volume according to the prismoidal formula.

69. Continuous Work. — If three parallel sections are measured, the areas being A, A' and A'', the perpendicular distance between A and A' being L, and that between A' and A'' being L', the total volume would be

$$\tfrac{1}{2}L(A + A') + \tfrac{1}{2}L'(A' + A''),$$

or

$$\tfrac{1}{2}LA + \tfrac{1}{2}(L + L')A' + \tfrac{1}{2}L'A''.$$

Hence, *if a section is common to two solids, multiply its area by one half the sum of the distances to the adjacent sections.*

70. Derivation of the General Formula.—The area $ABCDEFG$ is equivalent to the sum of the trapezoids $MABN$, $NBCO$, $OCDP$

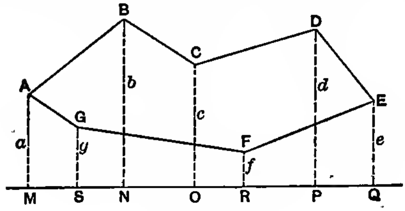

Fig. 46.

and $PDEQ$, diminished by the sum of the trapezoids $RFEQ$, $SGFR$ and $MAGS$, or

$$\tfrac{1}{2}MN\,(a+b)+\tfrac{1}{2}NO\,(b+c)+\tfrac{1}{2}OP\,(c+d)+\tfrac{1}{2}PQ\,(d+e)$$
$$-\tfrac{1}{2}RQ\,(e+f)-\tfrac{1}{2}SR\,(f+g)-\tfrac{1}{2}MS\,(g+a),$$

or $$\tfrac{1}{2}a\,(MN-MS)+\tfrac{1}{2}b\,(MN+NO)+\tfrac{1}{2}c\,(NO+OP)$$
$$+\tfrac{1}{2}d\,(OP+PQ)+\tfrac{1}{2}e\,(PQ-RQ)+\tfrac{1}{2}f\,(-RQ-SR)$$
$$+\tfrac{1}{2}g\,(-SR-MS),$$

or

$$\tfrac{1}{2}a\cdot SN+\tfrac{1}{2}b\cdot MO+\tfrac{1}{2}c\cdot NP+\tfrac{1}{2}d\cdot OQ+\tfrac{1}{2}e\,(-RP)$$
$$+\tfrac{1}{2}f\,(-SQ)+\tfrac{1}{2}g\,(-MR).$$

Multiplying this by $\tfrac{1}{2}L$ will give the volume of a right prism with this cross-section and with the length $\tfrac{1}{2}L$. These terms may be written by the use of Rule 14, Art. 72.

71. Definitions.—Passing a pencil around the area in the clockwise direction (from A to B to C to D to E to F to G to A),—of any three consecutive heights b, c and d, c is the **middle height**, d is the **forward height**, and b the **back height.**

72. The General Rule for the Average End Area Method. Rule 14.—*Passing around the solid clockwise, multiply each height by the horizontal distance between the forward and back heights; multiply the product by* $\tfrac{1}{4}L$, *giving the result the negative sign when the forward height is to the left of the back height; and add the results algebraically.*

This rule will answer for any polygonal cross-section, and it should always be used when the surface contains overhanging cliffs.

If h represents any height, and BFH is the horizontal distance between the back height B and the forward height F, the rule may be symbolized as follows:

73. Rule 14. The General Cross-Section. — Clockwise, for each height,

$$\tfrac{1}{4}Lh \cdot BFH \qquad (- \text{ when } F \text{ is left of } B).$$

74. Example. — Using the notes in Art. 24, where $L = 60$ feet, arrange the quantities in clockwise order, prefixing the last point and affixing the first, but paying no attention to the longitudinal edges:

	G'	A'	C'	D'	E'	F'	G'	A'	
	$\frac{30}{+35}$	$\frac{10}{+50}$	$\frac{40}{+60}$	$\frac{58}{+52}$	$\frac{74}{+55}$	$\frac{45}{+30}$	$\frac{30}{+35}$	$\frac{10}{+50}$	
$\frac{30}{+10}$	$\frac{20}{+30}$	$\frac{40}{+40}$	$\frac{60}{+34}$	$\frac{80}{+50}$	$\frac{90}{+35}$	$\frac{70}{+20}$	$\frac{50}{+18}$	$\frac{30}{+10}$	$\frac{20}{+30}$
H	A	B	C	D	E	F	G	H	A

Then the volume by Rule 14 is, in cubic yards,

	Terms.	+Vols.	−Vols.
For height at A	$\frac{1}{108} \times 60 \times 30 \times 10$	$166\frac{6}{9}$	
B	" $\times 40 \times 40$	$888\frac{8}{9}$	
C	" $\times 34 \times 40$	$755\frac{5}{9}$	
D	" $\times 50 \times 30$	$833\frac{3}{9}$	
E	" $\times 35 \times (-10)$		$194\frac{4}{9}$
F	" $\times 20 \times (-40)$		$444\frac{4}{9}$
G	" $\times 18 \times (-40)$		400
H	" $\times 10 \times (-30)$		$166\frac{6}{9}$
A'	" $\times 50 \times 10$	$277\frac{7}{9}$	
C'	" $\times 60 \times 48$	1600	
D'	" $\times 52 \times 34$	$982\frac{2}{9}$	
E'	" $\times 55 \times (-13)$		$397\frac{2}{9}$
F'	" $\times 30 \times (-44)$		$733\frac{3}{9}$
G'	" $\times 35 \times (-35)$		$680\frac{5}{9}$
	Totals =	$5504\frac{4}{9}$	$3016\frac{6}{9}$
		$3016\frac{6}{9}$	
	Volume =	$2487\frac{7}{9}$ cu. yds.	

75. Example. Second Method. — Using the same notes, but neglecting the longitudinal edges, we may first find the volume (Fig. 17 *a*) of $A_1ABCDEE_1\text{-}A_1'A'C'D'E'E_1'$, and then that of $A_1AHGFEE_1\text{-}A_1'A'G'F'E'E_1'$, finally subtracting the latter volume from the former. Prefix and affix to the notes new fractions which shall contain the measurements for A_1 and A_1' and for E_1 and E_1' respectively, so that the notes shall read as follows:

$$\begin{array}{cccccc} A_1' & A' & C' & D' & E' & E_1' \\ \frac{10}{0} & \frac{10}{+50} & \frac{40}{+60} & \frac{58}{+52} & \frac{74}{+55} & \frac{74}{0} \end{array}$$

$$\begin{array}{ccccccc} \frac{20}{0} & \frac{20}{+30} & \frac{40}{+40} & \frac{60}{+34} & \frac{80}{+50} & \frac{90}{+35} & \frac{90}{0} \\ A_1 & A & B & C & D & E & E_1 \end{array}$$

and, for the second solid,

$$\begin{array}{cccccc} A_1' & A' & G' & F' & E' & E_1' \\ \frac{10}{0} & \frac{10}{+50} & \frac{30}{+35} & \frac{45}{+30} & \frac{74}{+55} & \frac{74}{0} \end{array}$$

$$\begin{array}{ccccccc} \frac{20}{0} & \frac{20}{+30} & \frac{30}{+10} & \frac{50}{+18} & \frac{70}{+20} & \frac{90}{+35} & \frac{90}{0} \\ A_1 & A & H & G & F & E & E_1 \end{array}$$

Applying Rule 14 to these solids separately, we shall have, in cubic yards:

For the first solid.

Height.	Term.	Vols.
A	$\frac{1}{108} \times 60 \times 30 \times 20$	$333\frac{3}{9}$
B	" $\times 40 \times 40$	$888\frac{8}{9}$
C	" $\times 34 \times 40$	$755\frac{5}{9}$
D	" $\times 50 \times 30$	$833\frac{3}{9}$
E	" $\times 35 \times 10$	$194\frac{4}{9}$
A'	" $\times 50 \times 30$	$833\frac{3}{9}$
C'	" $\times 60 \times 48$	1600
D'	" $\times 52 \times 34$	$982\frac{2}{9}$
E'	" $\times 55 \times 16$	$488\frac{8}{9}$
	Total =	6910
		$4422\frac{2}{9}$
	Net Volume =	$2487\frac{7}{9}$ cu. yds.

For the second solid.

Height.	Term.	Vols.
A	$\frac{1}{108} \times 60 \times 30 \times 10$	$166\frac{6}{9}$
H	" $\times 10 \times 30$	$166\frac{6}{9}$
G	" $\times 18 \times 40$	400
F	" $\times 20 \times 40$	$444\frac{4}{9}$
E	" $\times 35 \times 20$	$388\frac{8}{9}$
A'	" $\times 50 \times 20$	$555\frac{5}{9}$
G'	" $\times 35 \times 35$	$680\frac{5}{9}$
F'	" $\times 30 \times 44$	$733\frac{3}{9}$
E'	" $\times 55 \times 29$	$886\frac{1}{9}$
	Total =	$4422\frac{2}{9}$

76. Application to the General Railroad Cross-Section. — By Rule 14, the volume of a right prism, with the cross-section shown in Fig. 47 and with the length $\frac{1}{2}L$, is

$$\tfrac{1}{4}Lf\cdot MC_1+\tfrac{1}{4}Lc\cdot OR+\tfrac{1}{4}Lg\cdot C_1N+\tfrac{1}{4}La\cdot PO+\tfrac{1}{4}Lb(-QR),$$

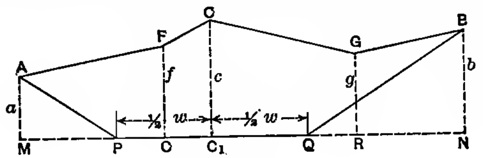

Fig. 47.

or

$$\tfrac{1}{4}Lf\cdot MC_1+\tfrac{1}{4}Lc\cdot OR+\tfrac{1}{4}Lg\cdot C_1N+\tfrac{1}{4}La\,(\tfrac{1}{2}w-C_1O)$$
$$+\tfrac{1}{4}Lb\,(\tfrac{1}{2}w-C_1R),$$

which will be given by Rule 15.

77. The Rule for the General Railroad Cross-Section. Rule 15. — (*a*) *For each height, except the extreme side heights: Multiply the height by the horizontal distance between the adjacent heights, and multiply the product by* $\frac{1}{4}L$.

(*b*) *For each extreme side height: Multiply the height by* $\frac{1}{2}w$ – *the distance out from the center to the height adjacent to the side height, and multiply the product by* $\frac{1}{4}L$.

(*c*) *Add algebraically the results of* (*a*) *and* (*b*).

This rule may be symbolized as follows:

78. Rule 15. The General Railroad Cross-Section.

For each height, except the extreme side heights,

$$\tfrac{1}{4}Lh\,.\,BFH.$$

For each extreme side height,

$$\tfrac{1}{4}Lh_s\,(\tfrac{1}{2}w-cah)\qquad(-\text{ when } cah>\tfrac{1}{2}w).$$

79. Rule 15 is Applicable to any railroad cross-section when one edge of the cross-section lies in the road-bed. If there are overhanging cliffs, it will be necessary to consider the sign of *BFH*. The distances C_1Q and C_1P are not necessarily equal; if unequal, the value of $\frac{1}{2}w$ used in Rule 15 must be the distance

C_1Q or C_1P that is measured towards the h_s that is being considered. The side slope ratios are not necessarily the same on the right and left. Note that cah is negative when the extreme side height and this adjacent height are on opposite sides of the center C_1 of the road-bed.*

80. Example. — Using the notes in Art. 32, but without paying attention to the longitudinal edges, the volume by Rule 15, when $L = 40$ feet, is, in cubic yards,

Height.	Terms.	+ Vols.	− Vol.
D	$\frac{1}{108} \times 40 \times 16 \times 17$	$100\frac{20}{27}$	
C	" $\times 17 \times 11$	$69\frac{7}{27}$	
E	" $\times 18 \times 22$	$146\frac{18}{27}$	
A	" $\times 10 \times 1$	$3\frac{19}{27}$	
B	" $\times 15 \times 2$	$11\frac{3}{27}$	
D'	" $\times 20 \times 25$	$185\frac{5}{27}$	
C'	" $\times 18 \times 38$	$253\frac{9}{27}$	
A'	" $\times 15 \times (-8)$		$44\frac{12}{27}$
B'	" $\times 10 \times 10$	$37\frac{1}{27}$	
	Totals =	$807\frac{1}{27}$	$44\frac{12}{27}$
		$44\frac{12}{27}$	
	Volume =	$762\frac{16}{27}$ cu. yds.	

81. The Area of the Three-Level Section, Fig. 48, is, since $PC_1 = C_1Q$,

$$\tfrac{1}{2}(a + b)\,\frac{w}{2} + \tfrac{1}{2}\,c\,(l + r).$$

Multiplying this area by $\frac{1}{2}L$ will give the volume of a right prism with this cross-section and with the length $\frac{1}{2}L$, the two terms being in accordance with Rule 16.

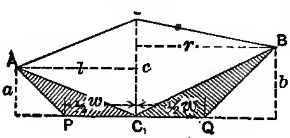

Fig. 48.

82. The Rule for the Three-Level Section. Rule 16. — (a) *Multiply the sum of the side heights by $\frac{1}{4}w$, and multiply the product by $\frac{1}{4}L$.*

(b) *Multiply the center height by the sum of the distances out from the center to the side heights, and multiply the product by $\frac{1}{4}L$.*

(c) *Add the results of (a) and (b).*

* For Rule 15 may be used when the height at C_1 is not measured.

This rule may be symbolized as follows:

83. Rule 16. Three-Level Sections.

$$\tfrac{1}{4} L\,(a+b)\,\frac{w}{2},$$
$$\tfrac{1}{4} L\,c\,(l+r).$$

84. Rule 16 is Applicable when either A or B, or both A and B, lie in the road-bed PQ. The side slope ratios on the right and left are not necessarily equal, but C_1Q must equal C_1P.

85. Example. — Using the notes in Art. 44, but without considering the longitudinal edges, the volumes by Rule 16 are, in cubic yards,

		Terms.	Vols.	Vols.
Sec. 3.	(a)	$\frac{1}{108} \times 100 \times 30 \times 7$	$194\frac{4}{9}$	
	(b)	" $\times 15 \times 59$	$819\frac{4}{9}$	
Sec. 2.	(a)	" $\times 30 \times 7$	$194\frac{4}{9}$	$2300\frac{25}{27}$
	(b)	" $\times 20 \times 59$	$1092\frac{16}{27}$	
	(a)	$\frac{1}{108} \times 60 \times 30 \times 7$	$116\frac{6}{9}$	
	(b)	" $\times 20 \times 59$	$655\frac{5}{9}$	
Sec. 1.	(a)	" $\times 42 \times 7$	$163\frac{3}{9}$	$2218\frac{8}{9}$
	(b)	" $\times 30 \times 77$	$1283\frac{3}{9}$	

86. The Area of the Three-Level Section, using the Grade Triangle. — When the side slopes AP and BQ are prolonged to their intersection at V, the area APC_1QBC is the sum of the triangles AVC and BVC diminished by the triangle PQV. But, if $\tan\theta = s$, and $VC_1 = c_0$, we have

$$c_0 = \frac{w}{2s}.$$

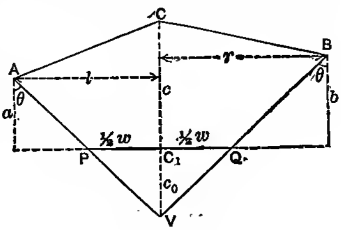

Fig. 49.

Hence the area is

$$\tfrac{1}{2}(c+c_0)(l+r) - \frac{w^2}{4s}.$$

Multiplying this by $\frac{1}{2}L$ will give the volume of the right prism whose cross-section is APC_1QBC and whose length is $\frac{1}{2}L$.

Note that $\dfrac{w^2}{4s} \times \frac{1}{2}L$ is the volume of the right prism whose cross-section is PQV and whose length is $\frac{1}{2}L$.

The required volume is given by Rule 17, $c + c_0$ being the **augmented height.**

87. The Rule for the Three-Level Section, using the Grade Prism. Rule 17. —*Multiply the augmented height by the sum of the distances out from the center to the side heights, and multiply the product by* $\frac{1}{4}L$. *From the result subtract* $w^2 L \div 8\,s$.

The rule may be expressed algebraically as follows:

88. Rule 17. Three-Level Sections, Special Case.

$$c_0 = w \div 2\,s.$$

$$E = \tfrac{1}{4} L\,(c + c_0)\,(l + r) - \frac{L\,w^2}{8\,s}.$$

89. The Application of Rule 17 is limited to cross-sections in which the surface intersects both side slopes. C_1P must equal C_1Q, and the angles C_1PA and C_1QB must be equal.

90. Example. — Using the notes in Art. 49, but without paying attention to the longitudinal edges, the volume by Rule 17 is, in cubic yards, since $L = 60$:

$$c_0 = w \div 2\,s = 14 \div 3 = 4\tfrac{2}{3},$$

Sec. 2. $c + c_0 = 24\frac{2}{3};\ \frac{1}{108} \times 60 \times 24\frac{2}{3} \times 59 =$ $808\frac{14}{27}$

Sec. 1. $c + c_0 = 34\frac{2}{3};\ \frac{1}{108} \times 60 \times 34\frac{2}{3} \times 77 =$ $1482\frac{26}{27}$

Sum $=$ $2291\frac{13}{27}$

$\frac{1}{27} \times \frac{Lw^2}{8\,s} = \frac{1}{216} \times 60 \times 14^2 \times \frac{2}{3}$

$= 36\frac{8}{27}\ ;\ 36\frac{8}{27} \times 2^* =$ $72\frac{16}{27}$

Volume $=$ $2218\frac{8}{9}$ cu. yds.

91. The Rule for Five-Level Sections *when two levels are taken above the edges of the road-bed*, Fig. 50. —By Rule 15, the volume of a right prism with this cross-section and with the length $\frac{1}{2}L$ is the sum of the three terms in

$$\textbf{Rule 18:}\quad \begin{cases} \tfrac{1}{4}\,L\,d\,l, \\ \tfrac{1}{4}\,L\,c\,w, \\ \tfrac{1}{4}\,L\,e\,r. \end{cases}$$

* Doubled, since it should be subtracted from each of the right prisms whose cross-sections are Sec. 1 and Sec. 2, respectively.

92. Example. — Using the notes in Art. 55, but neglecting the longitudinal edges, the volume by Rule 18 is, in cubic yards, since $L = 80$:

	Height.	Terms.		Vols.
Sec. 2.	d'	$\frac{1}{108} \times 80 \times 50 \times 100$	=	$3703\frac{19}{27}$
	c'	" $\times 40 \times 14$	=	$414\frac{22}{27}$
	e'	" $\times 30 \times 37$	=	$822\frac{6}{27}$
Sec. 1.	d	" $\times 70 \times 130$	=	$6740\frac{20}{27}$
	c	" $\times 60 \times 14$	=	$622\frac{6}{27}$
	e	" $\times 50 \times 70$	=	$2592\frac{16}{27}$
		Volume	=	$14896\frac{8}{27}$ cu. yds.

93. Rules for Level Sections, Fig. 51. — The volume of the

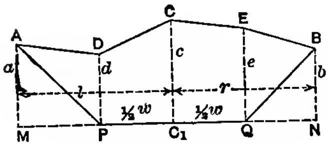

Fig. 50.

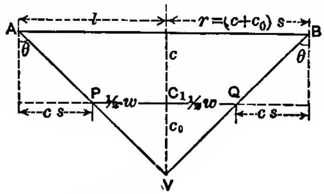

Fig. 51.

right prism with the cross-section APC_1QB and with the length $\frac{1}{2} L$ is

Rule 19 (a): $\frac{1}{4} L c (l + r + w)$.

Rule 19 (b): $\frac{1}{4} L c \cdot 2 (w + cs)$.

Rule 19 (c): $\frac{1}{4} L (c + c_0)^2 \cdot 2 s - \dfrac{L w^2}{8 s}$.

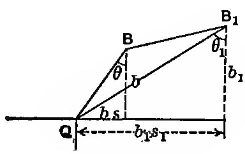

Fig. 52.

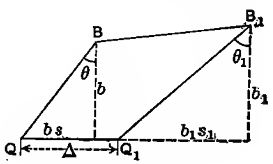

Fig. 53.

94. Rule when the Side Slope is Afterwards Flattened, Fig. 52. — The volume of the right prism with the cross-section BQB_1 and with the length $\frac{1}{2} L$ is

Rule 20: $\frac{1}{4} L \cdot b_1 \cdot b (s_1 - s)$.

95. Rules for Sidings, Fig. 53. — The volume of the right prism with the cross-section BQQ_1B_1 and with the length $\frac{1}{2}L$ is the sum of the two terms:

$$\textbf{Rule 21 (a):} \quad \begin{cases} \frac{1}{4} L (b + b_1) \Delta, \\ \frac{1}{4} L b_1 \cdot b (s_1 - s). \end{cases}$$

If $\theta = \theta_1$ so that $s = s_1$, Rule 21 (a) becomes

$$\textbf{Rule 21 (b):} \quad \frac{1}{4} L (b + b_1) \Delta.$$

96. Special Rule for Side-Hill Work, Triangular End Section, Fig. 54. — The volume of the right prism with the cross-section shown in Fig. 54 and with the length $\frac{1}{2}L$ is

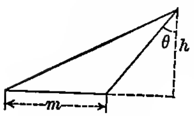

Fig. 54.

$$\textbf{Rule 22:} \quad \frac{1}{4} L h m.$$

THE AREA OF THE CROSS-SECTION.

97. The Area of the Cross-Section, in the different cases, may be found by the Rules in this chapter, provided that the factor $\frac{1}{2}L$ is omitted. Thus the area of the general cross-section will be found by passing around the section in the clockwise direction and multiplying each height by the horizontal distance between the forward and back heights, giving the product the negative sign when the forward height is to the left of the back height, adding the results algebraically, and dividing the sum by two.

CHAPTER III.

PRISMOIDAL CORRECTION WHEN THE CROSS-SECTIONS ARE DETERMINED BY LEVELS.

98. The Prismoidal Correction is a quantity which, subtracted algebraically from the volume between two end sections as determined by the average end area method, will give the volume that would be found by the use of the prismoidal formula.

Denoting the volume according to the prismoidal formula by P, and that according to the average end area method by E, and the correction by C, we have

$$\boldsymbol{P = E - C},$$

or

$$\boldsymbol{C = E - P}.$$

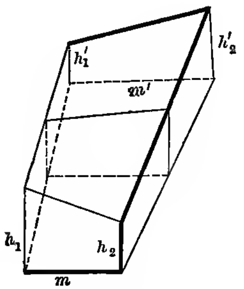

Fig. 55.

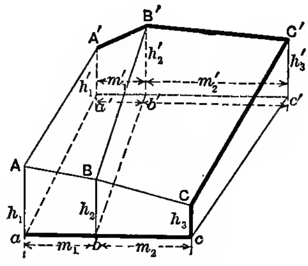

Fig. 56.

The volume of the solid shown in Fig. 55 is, according to the prismoidal formula, by Eq. (2),

$$P = \tfrac{1}{12} L\,[(2\,h_1 + h_1')\,m + (2\,h_1' + h_1)\,m' + (2\,h_2 + h_2')\,m + (2\,h_2' + h_2)\,m'];$$

and, by the average end area method,

$$E = \tfrac{1}{4} L\,[(h_1 + h_2)\,m + (h_1' + h_2')\,m'].$$

Hence

$$\boldsymbol{C = E - P = \tfrac{1}{12} L\,(m - m')\,[(h_1 + h_2) - (h_1' + h_2')]}\,. \qquad \textbf{(12)}$$

It is evident that $C = 0$ when either

$$m = m' \quad \text{or} \quad h_1 + h_2 = h_1' + h_2'.$$

99. Derivation of the Correction for the General Rule for the Average End Area Method. — Applying Eq. (12) to the solid below $ABB'A'$ (Fig. 56) and to that below $BB'C'C$, the total correction is

$$\begin{aligned} C &= \tfrac{1}{12} L\,(m_1 - m_1')\,(h_1 + h_2 - h_1' - h_2') \\ &\quad + \tfrac{1}{12} L\,(m_2 - m_2')\,(h_2 + h_3 - h_2' - h_3') \\ &= \tfrac{1}{12} L\,(h_1 - h_1')\,(m_1 - m_1') \\ &\quad + \tfrac{1}{12} L\,(h_2 - h_2')\,[\,m_1 + m_2) - (m_1' + m_2')] \\ &\quad + \tfrac{1}{12} L\,(h_3 - h_3')\,(m_2 - m_2'), \end{aligned}$$

so that the terms, in the correction, corresponding to the middle edge BB', are

$$\tfrac{1}{12} L\,(h_2 - h_2')\,[(m_1 + m_2) - (m_1' + m_2')] \quad . \quad . \quad (13)$$

leading to Rule 23.

100. General Rule for the Prismoidal Correction. Rule 23. — *For each longitudinal edge: From the height at the first end subtract the height at the other end; from the horizontal distance at the first end between the adjacent edges * subtract the horizontal distance at the other end between these adjacent edges; * and multiply the product of these results by* $\tfrac{1}{12} L$.

Using the notation in Appendix III, this rule may be symbolized as follows:

101. Rule 23. The General Prismoidal Correction. — Clockwise, for each edge,

$$\tfrac{1}{12} L\,(h - h')\,(bfe - bfe').$$

102. Example. — Using the notes in Art. 24, we have, in cubic yards,

Edge.	Terms.		+ Vols.	− Vols.
BA'	$\frac{1}{324} \times 60 \times (-10) \times (40 - 30)$	=		$18\frac{14}{27}$
CC'	" $\times (-26) \times (40 - 48)$	=	$38\frac{14}{27}$	
DD'	" $\times (-2) \times (30 - 34)$	=	$1\frac{13}{27}$	
EE'	" $\times (-20) \times (-10 + 13)$	=		$11\frac{3}{27}$
FF'	" $\times (-10) \times (-40 + 44)$	=		$7\frac{11}{27}$
GG'	" $\times (-17) \times (-40 + 15)$	=	$78\frac{19}{27}$	
HG'	" $\times (-25) \times (-30 + 20)$	=	$46\frac{8}{27}$	
AA'	" $\times (-20) \times (10 + 20)$	=		$111\frac{3}{27}$
	Totals	=	165	$148\frac{4}{27}$
			$148\frac{4}{27}$	
	C	=	$+16\frac{23}{27}$ cu. yds.	

From Art. 24, $P = 2470\frac{25}{27}$; from Art. 74, $E = 2487\frac{7}{9}$.

* Negative when that end of the forward edge is to the left of the corresponding end of the back edge.

103. Derivation of the Correction for the Rule for the General Railroad Solid. — The terms, in the expression for the correction, corresponding to the edge BB', are,

by Rule 23,

$$\tfrac{1}{12} L (h_s - h_s') [NQ - (- Q'N')],$$

or

$$\tfrac{1}{12} L (h_s - h_s') [(\tfrac{1}{2} w - cae) - (\tfrac{1}{2} w' - cae')],$$

leading to Rule 24.

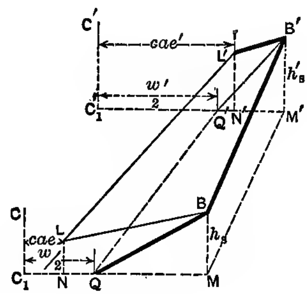

Fig. 57.

104. Correction for the Rule for the General Railroad Solid.* **Rule 24.** — (*a*) *For each edge except the extreme edges: From the height at the first end subtract the height at the other end; from the horizontal distance at the first end between the adjacent edges subtract the horizontal distance at the other end between these adjacent edges; and multiply the product of these results by* $\tfrac{1}{12} L$.

(*b*) *For each extreme edge: From the height at the first end subtract the height at the other end; at each end, from one half the width of the road-bed subtract the distance out from the center to the adjacent edge, and subtract the value at the second end from the value at the first end; and multiply the product of these results by* $\tfrac{1}{12} L$.

(*c*) *Add algebraically the results of* (*a*) *and* (*b*).

Using the notation in Appendix III, this rule may be symbolized as follows:

105. Rule 24. The General Railroad Solid Correction.

For each edge, except the extreme edges,

$$\tfrac{1}{12} L (h - h') (bfe - bfe').$$

For each extreme edge,

$$\tfrac{1}{12} L (h_s - h_s') [(\tfrac{1}{2} w - cae) - (\tfrac{1}{2} w' - cae')].$$

* See the foot-notes on page 22, and also Art. 31.

106. Example. — Using the notes in Art. 32, we have, in cubic yards,

Edge.		Terms.	+ Vols.	− Vols.
AD'	$\frac{1}{324}$	$\times\ 40 \times (-10)\ (11 - 7)$		$4\frac{76}{81}$
DD'	"	$\times\ (-\ 4)\ (17 - 18)$	$\frac{40}{81}$	
CC'	"	$\times\ (-\ 1)\ (11 - 38)$	$3\frac{27}{81}$	
EB'	"	$\times\ (+\ 8)\ (22 - 20)$	$1\frac{79}{81}$	
AA'	"	$\times\ (-\ 5)\ (-\ 10 + 8)$*	$1\frac{19}{81}$	
BB'	"	$\times\ (+\ 5)\ (2 + 10)$†	$7\frac{33}{81}$	
		Totals =	$14\frac{36}{81}$	$4\frac{76}{81}$
			$4\frac{76}{81}$	
		$\therefore\ C =$	$+9\frac{41}{81}$ cu. yds.	

From Art. 32, $P = 753\frac{7}{81}$; from Art. 80, $E = 762\frac{16}{27}$.

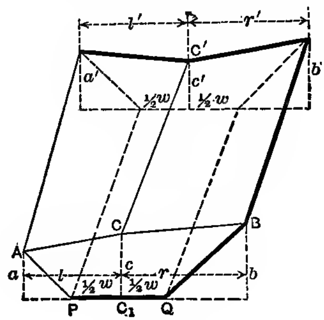

Fig. 58.

107. Derivation of the Correction for the Rule for Three-Level Sections with Two Warped Surfaces, the Width of the Road-Bed being Constant. — From Art. 40,

$$P = \tfrac{1}{4}\,L\,(a + b + a' + b')\,\frac{w}{2} + \tfrac{1}{12}\,L\,(2\,c + c')\,(l + r)$$
$$+ \tfrac{1}{12}\,L\,(2\,c' + c)\,(l' + r');$$

and from Rule 16,

$$E = \tfrac{1}{4}\,L\,(a + b)\,\frac{w}{2} + \tfrac{1}{4}\,L\,(a' + b')\,\frac{w}{2} + \tfrac{1}{4}\,L\,c\,(l + r)$$
$$+ \tfrac{1}{4}\,L\,c'\,(l' + r').$$

$$\therefore C = E - P = \tfrac{1}{12}\,L\,(c - c')\,[(l + r) - (l' + r')],$$

leading to Rule 25.

* $7 - 17 - (10 - 18)$. † $7 - 5 - (10 - 20)$.

108. Correction for the Rule for Three-Level Sections with Two Warped Surfaces, the Width of the Road-Bed being Constant. Rule 25. — *From the center height at the first end subtract the center height at the other end; from the sum of the distances out at the first end subtract the sum of the distances out at the other end; and multiply the product of these results by* $\frac{1}{12} L$.

109. Rule 25. Correction for Three-Level Sections.

$$\tfrac{1}{12} L\ (c - c')\ [(l + r) - (l' + r')].$$

NOTE 1.— When $c = c'$ or when $l + r = l' + r'$, the values of E and P are equal.

NOTE 2.— When the width of the road-bed is not constant, the correction is given by Rule 24, as follows:

$$C = \tfrac{1}{12} L \left(\frac{w}{2} - \frac{w'}{2}\right) (a - a' + b - b') + \tfrac{1}{12} L\, (c - c')\, [(l + r) - (l' + r')].$$

110. Rule 25 is Applicable to any of the modifications of the three-level section with which Rule 4 may be used (Art. 43).

111. Example. — Using the notes in Art. 44, the correction C for the volume between Sec. 1 and Sec. 2 is

$$\tfrac{1}{324} \times 60 \times (+\ 10) \times (77 - 59) = +\ 33\tfrac{1}{3} \text{ cu. yds.},$$

and that for the volume between Sec. 2 and Sec. 3 is

$$\tfrac{1}{324} \times 100 \times 5 \times (59 - 59) = 0.$$

From Arts. 44 and 85, for the first volume $P = 2185\tfrac{5}{9}$ and $E = 2218\tfrac{8}{9}$, and for the second volume $P = E = 2300\tfrac{25}{27}$.

112. Correction for the Rule for Five-Level Sections, two levels above the edge of the road-bed, with four warped surfaces, the width of the road-bed being constant, the side slopes either plane or warped (Fig. 59). — By Rule 24, Art. 105, the correction is

$$\begin{aligned} C = \tfrac{1}{12} L\, [&(d - d')\,(l - l') + (c - c')\,(w - w) + (e - e')\,(r - r') \\ &+ (a - a')\,(\overline{\tfrac12 w - \tfrac12 w} - \overline{\tfrac12 w - \tfrac12 w}) \\ &+ (b - b')\,(\overline{\tfrac12 w - \tfrac12 w} - \overline{\tfrac12 w - \tfrac12 w})] \\ = \tfrac{1}{12} L\, &(d - d')\,(l - l') + \tfrac{1}{12} L\, (e - e')\,(r - r'), \end{aligned}$$

or the sum of the terms

$$\textbf{Rule 26:} \quad \begin{cases} \tfrac{1}{12} L\ (d - d')\ (l - l'), \\ \tfrac{1}{12} L\ (e - e')\ (r - r'). \end{cases}$$

113. Example.—Using the notes in Art. 55, the correction is, in cubic yards,

$$\begin{aligned}
&\text{on the left,} \quad \tfrac{1}{324} \times 80 \times 20 \times 30 &&= 148\tfrac{4}{27}\\
&\text{on the right,} \quad \text{“} \qquad\;\; \times 20 \times 33 &&= 162\tfrac{26}{27}\\
&\qquad\qquad\qquad\qquad\quad \therefore\; C &&= 311\tfrac{1}{9} \text{ cu. yds.}
\end{aligned}$$

From Art. 55, $P = 14585\frac{5}{27}$; from Art. 92, $E = 14896\frac{8}{27}$.

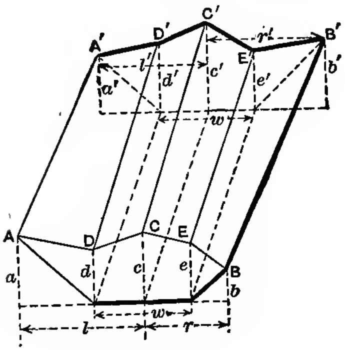

Fig. 59.

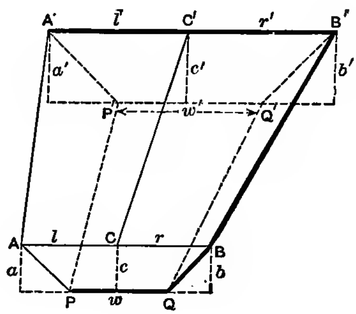

Fig. 60.

114. Correction for Level Sections. — When the width of the road-bed is different at the two ends (Fig. 60), the correction is

$$\textbf{Rule 27(a):}\quad \tfrac{1}{12} L\,(c - c')\,[(l + r + w) - (l' + r' + w')],$$

which is applicable when the side slopes are plane or warped, with equal or unequal side slope ratios.

When the width of the road-bed is constant, the correction is

$$\textbf{Rule 27(b):}\quad \tfrac{1}{12} L\,(c - c')\,[(l + r) - (l' + r')];$$

and, if also the side slopes are plane surfaces making the same angle with the road-bed, so that

$$l + r = w + 2\,cs \quad \text{and} \quad l' + r' = w + 2\,c's,$$

the correction becomes

$$\textbf{Rule 27(c):}\quad \tfrac{1}{12} L\,(c - c')^2 . 2\,s.$$

115. Correction when the Side Slope is Afterwards Flattened. — From Rule 9, Art. 58, the volume shown in Fig. 61 is

$$P = \tfrac{1}{12} L\,(2\,b_1 + b_1')\,b\,(s_1 - s) + \tfrac{1}{12} L\,(2\,b_1' + b_1)\,b'\,(s_1 - s);$$

and from Rule 20, Art. 94,

$$E = \tfrac{1}{4} L\,b_1 b\,(s_1 - s) + \tfrac{1}{4} L\,b_1' b'\,(s_1 - s);$$

and hence C $(= E - P)$ becomes

Rule 28: $\tfrac{1}{12} L \; s_1 - s)\,(b - b')\,(b_1 - b_1')$.

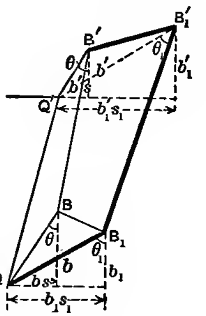

Fig. 61.

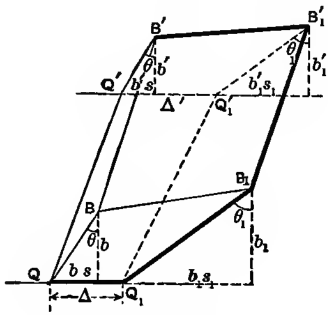

Fig. 62.

116. Correction for Sidings. — From Rule 10 (a), the volume of the solid shown in Fig. 62 is

$$P = \tfrac{1}{12} L\{[2\,(b + b_1) + (b' + b_1')]\,\Delta + [2\,(b' + b_1') + (b + b_1)]\,\Delta' + (2\,b_1 + b_1')\,b\,(s_1 - s) + (2\,b_1' + b_1)\,b'\,(s_1 - s)\};$$

and, from Rule 21(a),

$$E = \tfrac{1}{4} L\,[(b + b_1)\,\Delta + b_1 b\,(s_1 - s) + (b' + b_1')\,\Delta' + b_1' b'\,(s_1 - s)];$$

and hence C $(= E - P)$ becomes the sum of the two terms,

Rule 29(a): $\begin{cases} \tfrac{1}{12} L\,[(b + b_1) - (b' + b_1')]\,(\Delta - \Delta'), \\ \tfrac{1}{12} L\,(s_1 - s)\,(b - b')\,(b_1 - b_1'). \end{cases}$

If $s_1 = s$, this correction becomes

Rule 29(b): $\tfrac{1}{12} L\,[(b + b_1) - (b' + b_1')]\,(\Delta - \Delta')$.

If $s_1 = s$ and $\Delta' = \Delta$, the correction is zero.

117. Correction for the Side-Hill Solid with Triangular Ends and a Warped Surface. — From Rule 12, Art. 64,

$$P = \tfrac{1}{12} L [(2h + h')m + (2h' + h)m'];$$

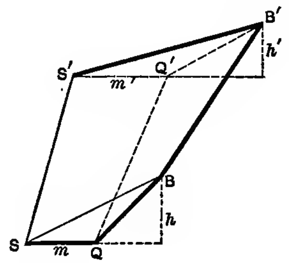

Fig. 63.

and from Rule 22, Art. 96,

$$E = \tfrac{1}{4} L (hm + h'm');$$

so that the correction C (= $E - P$) becomes

Rule 30: $\frac{1}{12} L (h - h')(m - m')$.

CHAPTER IV.

APPLICATION OF THE AVERAGE END AREA METHOD WHEN THE TRANSVERSE SLOPE OF THE GROUND IS MEASURED.

118. Preliminary Estimates of Quantities. — After the profile of the line of road has been measured and plotted, a grade line is assumed and drawn, and then the center heights of cut and fill are measured, equaling the vertical distances at the stations between the grade line and the profile. The customary cross-sectioning (Art. 27) could not be done in the field, since the grade line was not known, and therefore, owing to the lack of data, it is often assumed that the ground is level transversely, and the formulas for level sections are used.

To facilitate the computation, take a slip of paper divided on the same scale as the vertical scale of the profile, the division lines being parallel to the shorter side of the slip. Consider the distance from any division line to the base of the slip as representing the center height of a level section, and write on this division line the volume of a right prism with this level section and a length of 50 feet, or of 100 feet, as may be desired. Then, by placing the base of the slip on the grade line, the corresponding volume for a full section may be read by noting the point where the profile meets the slip. A separate slip will be necessary for each combination of width of road-bed and side slope ratio.

It is evident that the preceding method gives only a rough approximation to the truth, and hence the method of transverse slopes is adopted to secure increased accuracy at a small additional expenditure of labor. If the transverse slopes are measured (Art. 119) and the center height is afterwards determined from the profile and the grade line, the section really becomes a three-level section, so that the true area of the section, and the volume of the corresponding right prism, may be determined more closely than by assuming the surface to be level.

119. Method of Transverse Slopes. — The transverse slope of the ground on either side of the center line may be determined in one of two ways: (*a*) the angles ϕ' and ϕ'' (Fig. 64) may be measured with a clinometer, or (*b*) the rise or fall in going a given horizontal distance, say 10 or 20 or 30 or more feet, from the center may be measured, and then the tangent of the angle ϕ' or ϕ'' found by dividing the rise or fall by this horizontal distance.

We shall consider the area on one side of the center height C_1C, the resulting formulas being applicable to either side when the proper value of the angle ϕ is used. Multiplying this area by $\frac{1}{2}L$ will give the volume of a right prism with this cross-section and with the length $\frac{1}{2}L$.

If the center is in cut and the surface recedes from the road-bed as it leaves the center line, as shown on the right of C_1C in Fig. 64,

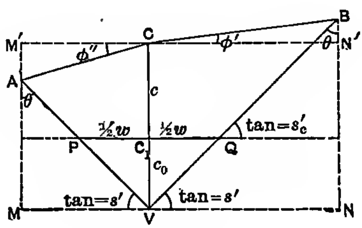

Fig. 64.

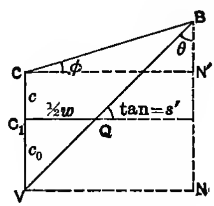

Fig. 65.

the corresponding area C_1CBQ of the cross-section will be entirely in cut; or, if the center is in fill and the surface recedes from the road-bed, the area will be entirely in fill.

If the surface is level, the area will be entirely in cut when the center is in cut, and entirely in fill when the center is in fill.

If the surface approaches the road-bed, several cases may arise:

(*a*) *The center in cut, and* $\tan \phi < c \div \frac{1}{2} w_c$.* — The surface does not intersect the road-bed, and the area is entirely in cut, Fig. 67.

* w_c = width of the road-bed in cut; w_f = width of the road-bed in fill; in practice, $w_c > w_f$.

(*b*) *The center in cut, and* $\tan\phi \overline{\underline{>}} c \div \frac{1}{2} w_c$. — The surface intersects the road-bed, but the area is entirely in cut unless $\tan\phi > c \div \frac{1}{2} w_f$ when some fill is necessary. Figs. 68 and 69.

(*c*) *The center in fill, and* $\tan\phi < c \div \frac{1}{2} w_c$. — The surface does not intersect the road-bed, and the area is entirely in fill. Fig. 70. See (*a*).

(*d*) *The center in fill, and* $\tan\phi \overline{\underline{>}} c \div \frac{1}{2} w_c$. — The surface intersects the road-bed, and the area is partly in fill and partly in cut (only fill when $\tan\phi = c \div \frac{1}{2} w_c$). Figs. 71, 72, 73 and 74.

120. When the Surface Recedes from the Road-Bed. — In Fig. 65, the area CC_1QB is equivalent to the area CVB less the area C_1QV. We have

$$\text{Area } CVB = \tfrac{1}{2}(c_0 + c)\, VN.$$

But $$NB = VN \tan NVB = VN \times s',$$

and also $$NB = VC + N'B = c_0 + c + VN \tan\phi.$$

Equating the two values of NB and solving,

$$VN = \frac{c_0 + c}{s' - \tan\phi},$$

and hence the area CVB is

$$\tfrac{1}{2}\,\frac{(c_0 + c)^2}{s' - \tan\phi}.$$

The area C_1QV is $\frac{1}{2} VC_1 \cdot C_1Q = \frac{1}{4} c_0 w = \frac{1}{8} w^2 s'$, and hence we have

$$\textbf{Area } \boldsymbol{CC_1QB} = \tfrac{1}{2}\,\frac{(c_0 + c)^2}{s' - \tan\phi} - \tfrac{1}{8} w^2 s',$$

this formula being true for both cut and fill, when the corresponding values of w, s' and c_0 are used.

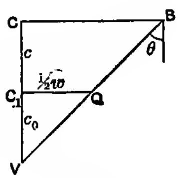

Fig. 66.

121. When the Surface is Level. — In Fig. 66, the area CC_1QB is

$$\tfrac{1}{2}(c_0 + c)^2 s - \tfrac{1}{8} w^2 s'$$

this formula being true for both cut and fill, when the corresponding values of w, s and c_0 are used.

122. The Center in Cut, the Surface Approaching the Road-Bed. — (*a*) *When the surface does not intersect the road-bed* ($\tan \phi < c \div \frac{1}{2} w_c$). — The area CC_1QB (Fig. 67) is equivalent to the area CVB less the area C_1QV. We have

$$\text{Area } CVB = \tfrac{1}{2}(c_0 + c)\, VN.$$

But $$NB = VN \tan NVB = VN \times s',$$

and also $$NB = NN' - BN' = c_0 + c - VN \tan \phi.$$

Equating the two values of NB and solving,

$$VN = \frac{c_0 + c}{s' + \tan \phi},$$

and hence the area CVB is

$$\tfrac{1}{2} \frac{(c_0 + c)^2}{s' + \tan \phi}.$$

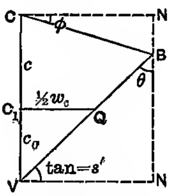

Fig. 67.

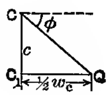

Fig. 68.

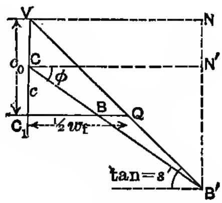

Fig. 69.

The area C_1QV is $\frac{1}{8} w^2 s'$, and hence we have

$$\textbf{Area } CC_1QB = \tfrac{1}{2} \frac{(c_0 + c)^2}{s' + \tan \phi} - \tfrac{1}{8} w^2 s'.$$

(*b*) *When the surface passes through the edge of the road-bed* ($\tan \phi = c \div \frac{1}{2} w_c$). — In Fig. 68,

$$\textbf{Area } CC_1Q = \tfrac{1}{2} \frac{c^2}{\tan \phi}.$$

(*c*) *When the surface intersects the road-bed* ($\tan \phi > c \div \frac{1}{2} w_c$). — (1) The area in cut in Fig. 69 is

$$\textbf{Area } CC_1B = \tfrac{1}{2} \frac{c^2}{\tan \phi}.$$

(2) There will be an area in fill when $\tan \phi > c \div \frac{1}{2} w_f$, found as follows:

The area BQB' (Fig. 69) is equivalent to the sum of the areas $VB'C$ and CBC_1 less the area VQC_1. We have

$$\text{Area } VB'C = \tfrac{1}{2}(c_0 - c)\, VN.$$

But $\qquad B'N = VN \tan NVB' = VN \times s',$

and also $\qquad B'N = CV + B'N' = c_0 - c + VN \tan \phi.$

Equating the two values of $B'N$ and solving,

$$VN = \frac{c_0 - c}{s' - \tan \phi},$$

and hence the area $VB'C$ is

$$\frac{1}{2} \frac{(c_0 - c)^2}{s' - \tan \phi}.$$

The area $CBC_1 = \frac{1}{2} \frac{c^2}{\tan \phi}$, and the area $VC_1Q = \frac{1}{8} w^2 s'$. Hence we have

$$\textbf{Area } BQB' = \frac{1}{2} \frac{(c_0 - c)^2}{s' - \tan \phi} + \frac{1}{2} \frac{c^2}{\tan \phi} - \frac{1}{8} w^2 s'$$

where the values of w, s' and c_0 are those for a fill.

123. The Center in Fill, the Surface Approaching the Road-Bed. — (*a*) *When the surface does not intersect the road-bed*

Figs. 70, 71, 72, 73, 74.

($\tan \phi < c \div \frac{1}{2} w_c$). — The formula of Art. 122(*a*) is applicable to Fig. 70.

(*b*) *When the surface passes through the edge of the road-bed for cuts* ($\tan \phi = c \div \frac{1}{2} w_c$). — The half cross-section in fill in Fig. 71 is given by the formula of Art. 122(*a*). There is no cut.

(*c*) *When the surface cuts the road-bed between the edges of the road-bed for cut and that for fill* ($c \div \frac{1}{2} w_c < \tan \phi < c \div \frac{1}{2} w_f$). — The area in fill in Fig. 72 is given by the formula of Art. 122(*a*). The area in cut is given by the last formula in (2) of Art. 122(*c*).

(*d*) *When the surface passes through the edge of, or intersects, the road-bed for fill* ($\tan \phi \geqq c \div \frac{1}{2} w_f$). — The area in fill in Figs. 73 and 74 is given by the formula in (1) of Art. 122(*c*), and that in cut by the last formula in (2) of Art. 122(*c*).

Note. — In applying the formulas of Art. 122 when the center is in fill, the values of w, s' and c_0 should be those for a fill except in the last formula in (2) of Art. 122(*c*) where they should be those for a cut.

124. Summary. — Let the center height in cut be h_c, the volume in cut be V_c, and the width of the road-bed in cut be w_c, the subscript c denoting that the quantities are in cut. Similarly let h_f, V_f and w_f be the corresponding quantities in fill. Then the height in cut h_c and the volume in cut V_c have *like* subscripts c, and the height in cut h_c and the volume in fill V_f have *unlike* subscripts c and f. In this sense let

V_l = volume with subscript *like* that of the center height,
V_u = volume with subscript *unlike* that of the center height,
w_l = width of road-bed with subscript *like* that of the center height,
w_u = width of road-bed with subscript *unlike* that of the center height,

and so also with the other letters.

I. Surface recedes * from the road-bed.†

$$V_l = \frac{L}{4}\,\frac{(c_0 + c)^2}{s_l' - \tan\phi} - \tfrac{1}{16}\,L w_l^2 s_l'. \quad \left[c_0 = \frac{w_l}{2\,s_l}\right]. \quad V_u = 0.$$

II. Surface level.†

$$V_l = \tfrac{1}{4}\,L\,(c_0 + c)^2\,s_l - \tfrac{1}{16}\,L w_l^2 s_l'. \quad \left[c_0 = \frac{w_l}{2\,s_l}\right]. \quad V_u = 0.$$

III. Surface approaches† the road-bed.‡

(*a*) $\tan\phi < c \div \tfrac{1}{2}\,w_l$;

$$V_l = \frac{L}{4}\,\frac{(c_0 + c)^2}{s_l' + \tan\phi} - \tfrac{1}{16}\,L w_l^2 s_l'. \quad \left[c_0 = \frac{w_l}{2\,s_l}\right].$$

(*b*) $\tan\phi \overline{>} c \div \tfrac{1}{2}\,w_l$; $\quad V_l = \dfrac{L}{4}\,\dfrac{c^2}{\tan\phi}$.

(*c*) $\tan\phi > c \div \tfrac{1}{2}\,w_u$;‡

$$V_u = \frac{L}{4}\,\frac{(c_0 - c)^2}{s_u' - \tan\phi} + \frac{L}{4}\,\frac{c^2}{\tan\phi} - \tfrac{1}{16}\,L w_u^2 s_u'. \quad \left[c_0 = \frac{w_u}{2\,s_u}\right].$$

125. Example 1. — The road-bed in cut being 18 feet wide, and in fill 14 feet, and the side slope ratio 1 to 1 in cut ($s_c = 1$)

* In going away from the center C.
† $\tfrac{1}{16}\,Lw^2s' = \tfrac{1}{4}\,G$.
‡ $V_u = 0$ in Cases I and II, and also in Case III unless $\tan\phi > c \div \tfrac{1}{2}\,w_u$.

and $1\frac{1}{2}$ to 1 in fill ($s_f = \frac{3}{2}$), let the notes for a cross-section be

Slope.	Center Height.	Slope.
$-\dfrac{3.0}{10}$	$+30$	$+\dfrac{0.25}{10}$

the minus sign of the slope indicating that the ground goes down from the center, and the positive that it goes up. Let the distance to the adjacent section be 60 feet, so that $L = 60$.

The center is in cut. On the right the ground rises and hence recedes from the road-bed; therefore we use the formula of Case I. On the left the ground approaches the road-bed, so that Case III is applicable; then

$$c \div \tfrac{1}{2} w_l = 30 \div 9 = 3.33 +, \text{ and } \tan\phi = \frac{3.0}{10} = 0.3,$$

so that we use the formula of Case III(a); also

$$c \div \tfrac{1}{2} w_u = 30 \div 7 = 4.28 +, \text{ and } \tan\phi = 0.3,$$

so that Case III(c) does not apply, and hence there is no volume in fill.

Since, for the volume in cut, $c_0 = w_c \div 2\,s_c = 18 \div 2 = 9$, we have:

On left, $s_l' + \tan\phi = 1 + 0.3 = 1.3$, $c_0 + c = 39$;

$$\therefore \frac{1}{27} \; \frac{60}{4} \; \frac{39^2}{1.3} = 650 \text{ cu. yds.*}$$

On right, $s_l' - \tan\phi = 1 - 0.025 = 0.975$, $c_0 + c = 39$;

$$\therefore \frac{1}{27} \; \frac{60}{4} \; \frac{39^2}{0.975} = 866\tfrac{2}{3} \text{ cu. yds.*}$$

$$1516\tfrac{2}{3}$$

$$2 \times \tfrac{1}{4} G_l = 2 \times \tfrac{1}{27} \times \tfrac{1}{16} \times 60 \times 18^2 \times 1 = 90 \text{ cu. yds.*}$$

$$\therefore \text{Volume}\dagger = 1426\tfrac{2}{3} \text{ cu. yds.}$$

126. Example 2. — The road-bed in cut being 28 feet wide, and in fill 20 feet, and the side slope ratio 1 to 1 in cut ($s_c = 1$) and $1\frac{1}{2}$ to 1 in fill ($s_f = \frac{3}{2}$), let the notes for a cross-section be

Slope.	Center Height.	Slope.
$-\dfrac{6.0}{10}$	$+2.0$	$+\dfrac{5.0}{10}$

the distance L to the adjacent section being 60 feet.

* The factor $\frac{1}{27}$ is introduced to reduce to cubic yards. These results may be obtained mechanically by the use of the Crockett Volume Slide Rule.

† Of a right prism with the given cross-section and length $\frac{1}{2} L = 30$ feet.

The center is in cut. On the right the ground rises and therefore recedes from the road-bed; hence we use the formula of Case I. On the left the ground approaches the road-bed, so that Case III is applicable; then

$$c \div \tfrac{1}{2} w_l = 2 \div 14 = 0.14 +, \text{ and } \tan \phi = 0.6,$$

so that we use the formula of Case III(b) to find V_l, the volume in cut; also

$$c \div \tfrac{1}{2} w_u = 2 \div 10 = 0.2, \text{ and } \tan \phi = 0.6,$$

so that Case III(c) is applicable, giving a volume V_u which in this example will be in fill.

For the volume in cut, $s_l' = s_c' = 1$ and $c_0 = 28 \div 2 = 14$; and for that in fill, $s_u' = s_f' = \frac{2}{3}$ and $c_0 = 20 \div 3 = 6.7$ approximately. Hence we have, for the volume in cut,

On left, $\quad \dfrac{1}{27} \; \dfrac{60}{4} \; \dfrac{2^2}{0.6} \quad = \quad 3\frac{19}{27}$ cu. yds.*

On right, $s_l' - \tan \phi = 1 - 0.5 = 0.5,$

$$c_0 + c = 16;$$

$$\therefore \quad \frac{1}{27} \; \frac{60}{4} \; \frac{16^2}{0.5} \quad = \quad 284\tfrac{4}{9} \text{ cu. yds.*}$$

$$288\tfrac{4}{27}$$

$$\tfrac{1}{4} G_l = \tfrac{1}{27} \times \tfrac{60}{16} \times 28^2 \quad = \quad 108\tfrac{8}{9} \text{ cu. yds.*}$$

$$\therefore \text{ Volume† in cut} = 179\tfrac{7}{27} \text{ cu. yds.}$$

For the volume on the left that is in fill (V_u), by Case III(c),

$$s_u' - \tan \phi = \tfrac{2}{3} - 0.6 = 0.667 - 0.6 = 0.067,$$

$$c_0 - c = 6.7 - 2 = 4.7;$$

$$\therefore \quad \frac{1}{27} \; \frac{60}{4} \; \frac{4.7^2}{0.067} = 183.2 \text{ cu. yds.*}$$

$$\frac{1}{27} \; \frac{60}{4} \; \frac{2^2}{0.6} \quad = \quad 3.7 \text{ cu. yds.*}$$

$$186.9$$

$$\tfrac{1}{4} G_u = \tfrac{1}{27} \times \tfrac{60}{16} \times 20^2 \times 0.667 = 37.1 \text{ cu. yds.*}$$

$$\therefore \text{ Volume† in fill} = 149.8 \text{ cu. yds.}$$

* The factor $\frac{1}{27}$ is introduced to reduce to cubic yards. These results may be obtained mechanically by the use of the Crockett Volume Slide Rule.

† Of a right prism with the given cross-section and length $\frac{1}{2} L = 30$ feet.

127. The Position of the Slope Stake in these cases is as follows:

I. *When the surface recedes from the road-bed.*

$$\text{Distance out} = \frac{c_0 + c}{s_l' - \tan\phi};$$

$$\text{Elevation} = \frac{(c_0 + c)}{s_l' - \tan\phi}\, s_l' - c_0. \qquad \left[c_0 = \frac{w_l}{2\, s_l}\right].$$

II. *When the surface is parallel to the road-bed.*

$$\text{Distance out} = (c_0 + c)\, s_l\,; \text{ Elevation} = c. \qquad \left[c_0 = \frac{w_l}{2\, s_l}\right].$$

III. *When the surface approaches the road-bed.*

(*a*) $\tan\phi < c \div \frac{1}{2}\, w_l$; Distance out $= \dfrac{c_0 + c}{s_l' + \tan\phi}$;

$$\text{Elevation} = \frac{(c_0 + c)}{s_l' + \tan\phi}\, s_l' - c_0. \qquad \left[c_0 = \frac{w_l}{2\, s_l}\right].$$

(*b*) $\tan\phi \geqq c \div \frac{1}{2}\, w_l$; Distance out $= \dfrac{c}{\tan\phi}$; Elevation $= 0$.

(*c*) $\tan\phi > c \div \frac{1}{2}\, w_u$; Distance out* $= \dfrac{c_0 - c}{s_u' - \tan\phi}$;

$$\text{Elevation*} = \frac{(c_0 - c)}{s_u' - \tan\phi}\, s_u' - c_0. \qquad \left[c_0 = \frac{w_u}{2\, s_u}\right].$$

128. Example.—The road-bed in cut being 28 feet wide, and in fill 20 feet wide, the side slope ratio in cut 1 to 1 and in fill $1\frac{1}{2}$ to 1 (horizontal ÷ vertical), let the notes for a section be

Slope.	Center Height.	Slope.
$-\dfrac{6.0}{10}$	$+\,2.0$	$+\dfrac{5.0}{10}$

(1) *On the right.*—The center is in cut, and the surface slopes upwards and hence recedes from the road-bed, so that Case I is applicable; $c_0 = \frac{1}{2}\, w = 14$, $s' = 1$, $\tan\ \phi = 0.5$; $c_0 + c = 16$, $s' - \tan\phi = 0.5$.

$$\therefore \text{Distance out} = \frac{16}{0.5} = 32.0 \text{ ft.}$$

$$\text{Elevation} = 32.0 \times 1 - 14 = +\,18.0 \text{ ft.}$$

* Note that the point B' in Fig. 69 is on the opposite side of the road-bed from C.

(2) *On the left.* — The center is in cut, and the surface slopes downwards and hence approaches the road-bed, so that Case III is applicable. Then $\frac{1}{2}\,w_l = 14$, and therefore

$$c \div \tfrac{1}{2}\,w_l = 2 \div 14 = 0.14 +, \text{ and } \tan\phi = 0.6,$$

so that Case III(b) is used.

$$\therefore \text{ Distance out } = \frac{2}{0.6} = 3\tfrac{1}{3} \text{ ft. to where elevation} = 0,$$

that is, to the point B in Fig. 69.

Moreover, since $\frac{1}{2}\,w_u = 10$,

$$c \div \tfrac{1}{2}\,w_u = 2 \div 10 = 0.2 \text{ and } \tan\phi = 0.6,$$

so that Case III(c) is applicable. Since $s_u = \frac{2}{3}$, $c_0 = 10 \div \frac{3}{2} = 6\frac{2}{3}$,

$$c_0 - c = 4\tfrac{2}{3} = \tfrac{14}{3},\quad s_u' - \tan\phi = \tfrac{2}{3} - \tfrac{6}{10} = \tfrac{1}{15}.$$

$$\therefore \text{ Distance out } = \frac{\frac{14}{3}}{\frac{1}{15}} = 70.0 \text{ ft.}$$

$$\text{Elevation} = 70.0 \times \tfrac{2}{3} - 6\tfrac{2}{3} = -\,40.0 \text{ ft.},$$

the negative sign being used because the point (B' in Fig. 69) is in the fill.

CHAPTER V.

THE CORRECTION FOR CURVATURE.

129. The Cross-Sections. — When the road-bed is curved, it is customary to measure the cross-sections in radial planes, instead of at right angles to the chord, then computing the volume as though the solid were straight, and applying a correction for curvature.

Thus, in Fig. 75, where P is the center of the curve, the arcs C_1C_1', LL' and RR' are the center and the side lines, respectively, of the road-bed, the radius PC_1 is R, and the length of the chord C_1C_1' is L. The cross-sections are measured in the radial planes PC_1 and PC_1', A, B and A', B' being the points where the side slopes meet the surface. Hence, if AA' and BB' are the lines cut from the surface by the side slopes, the required volume is bounded horizontally by $ABB'A'$.

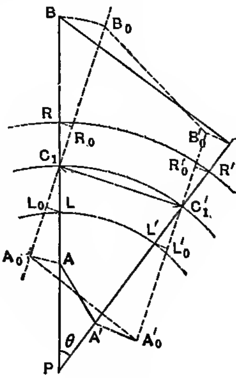

Fig. 75.

130. The Two Methods of Computing the Correction for Curvature. — It is evident from the conditions arising in practice that the positions of the lines AA' and BB' can be determined only approximately, so that an approximate value of the correction for curvature is all that is attainable. Two methods of determining this correction for curvature are in use, the resulting correction not necessarily being the same in the two cases.

First. — Each radial cross-section is revolved about the vertical line through the center of the road-bed until it is at right angles to the chord, and the correction for curvature is assumed to be the sum of the volumes described by the portions of the cross-sections outside the center line of the road-bed (that is, by C_1B and $C_1'B'$) diminished by the sum of the volumes described by

the portions of the cross-sections inside the center line of the road-bed (that is, by C_1A and $C_1'A'$). This method is considered in Art. 131 *et seq.*

Second. — The longitudinal edges AA' and BB' are assumed to be curved lines, determined by the property that the change in the elevation above the road-bed, and that in the corresponding distance from the center of the curve, shall be proportional to the change in the angular distance from a measured cross-section. The volume is then determined by the principle that the volume described by any moving area equals the product of the area by the distance, at right angles to the area, traversed by the center of gravity. This method is considered in Art. 135 *et seq.*

THE FIRST METHOD.

131. Three-Level Sections. — Let two planes $A_0C_1B_0$ and $A_0'C_1'B_0'$, Fig. 75, be passed perpendicular to the chord L, and let the plane sections AB and $A'B'$ be revolved about the vertical lines through C_1 and C_1' respectively until AB coincides with A_0B_0 and $A'B'$ with $A_0'B_0'$. Then the volume bounded by $A_0B_0B_0'A_0'$ is computed as a straight solid with the length L and with the given cross-sections; and it is assumed that (to find the volume * $ABB'A'$) the correction to be added at the end C_1 is the volume BC_1B_0 less the volume AC_1A_0, and at the end C_1' it is the volume $B'C_1'B_0'$ less the volume $A'C_1'A_0'$†.

In Fig. 75, let the angle $C_1PC_1' = \theta$; then $A_0C_1A = B_0C_1B = A_0'C_1'A' = B_0'C_1'B' = \frac{1}{2}\theta$. Also, $\sin\frac{1}{2}\theta = \frac{1}{2}\frac{L}{R}$, and therefore, since θ is small, approximately $\theta = \frac{L}{R}$.

In Fig. 76, let $ACBRL$ be the cross-section made by the vertical plane PC_1 (Fig. 75), A being the point inside the curve and B the point outside. Through C draw CA_1 so that the angle $C_1CA_1 = C_1CA$. Then, if the cross-section be revolved about C_1C through the angle $\frac{1}{2}\theta$, the correction at C_1 will be the volume generated by CC_1RB less the volume generated by CC_1LA, and hence it equals the volume generated by CA_1B. This volume equals the portion of a frustum of a cone described by FA_1BG diminished

* The volumes are bounded horizontally by the figures named.

† Field-Book for Railroad Engineers, by John B. Henck, 1854.

by the sum of the portions of the cones described by CBG and CA_1F, or

$$\tfrac{1}{6}(b-a)\left[\tfrac{1}{2}r^2\frac{\theta}{2}+\tfrac{1}{2}l^2\frac{\theta}{2}+4\cdot\tfrac{1}{2}\left(\frac{r+l}{2}\right)^2\frac{\theta}{2}\right]$$
$$-\tfrac{1}{3}(b-c)\tfrac{1}{2}r^2\frac{\theta}{2}-\tfrac{1}{3}(c-a)\tfrac{1}{2}l^2\frac{\theta}{2},$$

or
$$\tfrac{1}{12}\theta(r+l)[c(r-l)+bl-ar].$$

But $\quad bl-ar=b(\tfrac{1}{2}w+as)-a(\tfrac{1}{2}w+bs)=\tfrac{1}{2}w(b-a),$

and $\quad \theta=L\div R.$

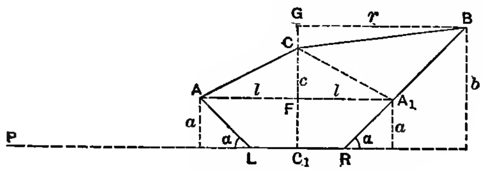

Fig. 76.

Hence the correction, *at one end of the solid*, becomes

$$\textbf{Corr.}=\frac{L}{12R}(r+l)[c(r-l)+\tfrac{1}{2}w(b-a)]\quad.\quad.\quad(14)$$

Eq. (14) may be written

$$\text{Corr.}=\frac{L}{12R}\left[c(r+l)(r-l)+\tfrac{1}{2}w(r+l)\frac{r-l}{s}\right]^*$$
$$=\frac{L(r-l)}{12R}\left[c(r+l)+\frac{w}{2s}(w+as+bs)\right]$$
$$=\frac{r-l}{3R}\ \frac{L}{4}\left[c(r+l)+\frac{w^2}{2s}+\frac{w}{2}(a+b)\right].$$

$$\therefore\ \textbf{Corr.}=\frac{r-l}{3R}(E_2+\tfrac{1}{2}G)\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\quad(15)$$

where E_2 is the volume of a right prism with the given cross-section $ACBRL$ and length $\tfrac{1}{4}L$, and G is the volume of the grade prism with length L.

To find the volume between the radial planes PB and PB', find separately, by Eq. (14) or by Eq. (15), the correction at each radial plane, and add them algebraically to the "straight volume."

* Or, correction $=\dfrac{L(r+l)(r-l)}{12R}(c+c_0)$, from which Eq. (15) may be easily derived.

The total correction at any station, or at any sub-chord station is found by Eq. (14) or by Eq. (15), provided that L is the sum of the lengths of the adjacent chords.

The total correction at any tangent point is found by Eq. (14) or by Eq. (15), provided that L is the length of the adjacent chord.

If, in Eq. (15), E_2 and G are expressed in cubic yards, the correction will also be in cubic yards.

NOTE 1.— In using Eq. (14) or Eq. (15), remember that r is always the distance out to the slope stake on the outside of the curve, and that the correction is to be added algebraically to the straight volume.

NOTE 2.— When the cross-section is irregular, an approximate value of the correction may be determined by first finding the dimensions of a two-level or of a three-level cross-section whose area equals that of the given cross-section, by the methods given in Appendix II, and then using this equivalent section in the formulas of Art. 131.

132. Special Case. — *When the cross-section of the surface is a single straight line.* — (1) If, in Fig. 76, ACB is a single straight line, the correction will still be given by Eq. (14) or by Eq. (15).

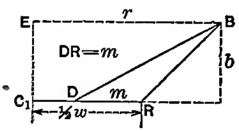

Fig. 77.

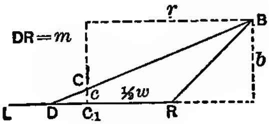

Fig. 78.

(2) When the cross-section is like either Fig. 77 or Fig. 78, the correction is

$$\frac{L\,b\,m(r + w - m)}{12\,R},$$

or

$$\frac{r + w - m}{3\,R}\,E_2,$$

where E_2 is the volume of a right prism with the cross-section BDR and length $\frac{1}{2}L$.*

* The correction is added algebraically to E_2 when B is the outside slope stake, and subtracted algebraically when B is the inside slope stake.

133. Special Case. — When the cross-section is like Fig. 79 or Fig. 80, the correction is

$$\frac{L r^2 (c + c_0)}{12 R} - \frac{L w^2 c_0}{48 R},$$

or

$$\frac{r}{3 R} (E_2 + \tfrac{1}{4} G) - \frac{w}{6 R} (\tfrac{1}{4} G)$$

where E_2 is the volume of a right prism with the cross-section CC_1RB and length $\frac{1}{2} L$. See the foot-note on page 76.

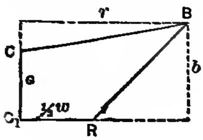

Fig. 79.

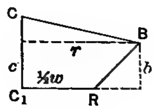

Fig. 80.

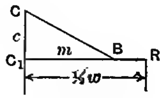

Fig. 81.

134. Special Case. — When the cross-section is like Fig. 81, the correction is

$$\frac{L m^2 c}{12 R}$$

or

$$\frac{m}{3 R} E_2$$

where E_2 is the volume of a right prism with the cross-section CC_1B and length $\frac{1}{2} L$. See the foot-note on page 76.

THE SECOND METHOD.

135. Three-Level Sections. — In Fig. 82, which is a horizontal projection, let P be the center of the curve $C_1'C_mC_1''$, $L'LL''$ and $R'RR''$ the sides of the road-bed, and $A'AA''$ and $B'BB''$ the curves in which the side slopes intersect the surface of the earth. The end sections of the curved prismoid

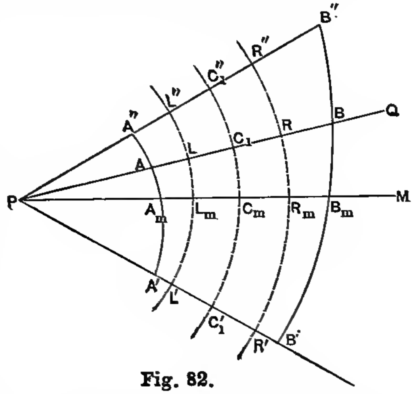

Fig. 82.

are $A'L'C_1'R'B'$ and $A''L''C_1''R''B''$, PM is a vertical plane through the middle point C_m of $C_1'C_mC_1C_1''$, while ALC_1RB is the section

cut by any radial vertical plane PQ. This section, made by the plane PQ, will be in the form ALC_1RBCA, shown in Fig. 83, where the triangle LC_1RV is formed by prolonging the side slopes AL and BR to their intersection with CC_1.

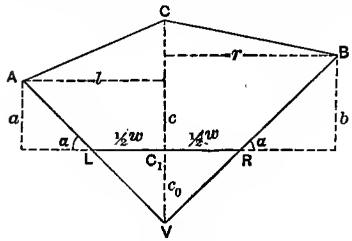

Fig. 83.

In this method it is assumed that the change in elevation, above the road-bed, along the lines $A'AA''$, $C_1'C_mC_1''$ and $B'BB''$, is directly proportional to the angular distance, about P, from the end section $A'L'C_1'R'B'$. The derivation of the formula depends upon the fact that the volume described by any moving area equals the product of the area by the distance traversed by its center of gravity, at right angles to the area. We shall first consider the volume corresponding to the complete section $AVBC$, deducting from the final result the volume corresponding to the triangle LVR.

Let R be the radius of the curve $C_1'C_mC_1''$, and L the length of the arc $C_1'C_mC_1''$; a', b', c', l', r' the side and center heights and the distances out in the section $PA'B'$; a'', b'', c'', l'', r'' those in the section $PA''B''$; a, b, c, l, r in PAB; and a_m, b_m, c_m, l_m, r_m in PA_mB_m. Then

$$A_m = \tfrac{1}{2}(a'' + a'), \quad b_m = \tfrac{1}{2}(b'' + b'), \quad c_m = \tfrac{1}{2}(c'' + c').$$
$$l_m = \tfrac{1}{2}(l'' + l'), \quad r_m = \tfrac{1}{2}(r'' + r').$$
$$\text{Let } D_a = a'' - a', \quad D_b = b'' - b', \quad D_c = c'' - c',$$
$$D_l = l'' - l', \quad D_r = r'' - r',$$

and let $C_mC_1 = x$, measured along the curve $C_1'C_mC_1''$ from the mid-section.

$$\therefore \quad a = a_m + \frac{D_a}{L}x, \quad b = b_m + \frac{D_b}{L}x, \quad c = c_m + \frac{D_c}{L}x,$$
$$l = l_m + \frac{D_l}{L}x, \quad r = r_m + \frac{D_r}{L}x.$$

The perpendicular distance from any line in the plane of a triangle to its center of gravity is one third of the sum of the distances from the line to the three vertices of the triangle. Hence,

considering a vertical line through P, the distance to the center of gravity of AVC is

$$\tfrac{1}{3}(3R - l),$$

and that to the center of gravity of BVC is

$$\tfrac{1}{3}(3R + r).$$

We also have

$$\text{Area } AVC = \tfrac{1}{2}\, l\,(c_0 + c)$$

and

$$\text{Area } BVC = \tfrac{1}{2}\, r\,(c_0 + c).$$

Hence, denoting the area of $AVBC$ by A, we have

$$A = \tfrac{1}{2}(r + l)(c_0 + c).$$

If ρ is the horizontal distance from P to the center of gravity of $AVBC$,

$$\begin{aligned}
\rho A &= \tfrac{1}{6}[(3R - l)\,l + (3R + r)\,r](c_0 + c)\\
&= \tfrac{1}{6}[3R(c_0 + c)(r + l) + (c_0 + c)(r^2 - l^2)]\\
&= \tfrac{1}{6}[6RA + (c_0 + c)(r + l)(r - l)]
\end{aligned}$$

$$= RA + \tfrac{1}{6}(c_0 + c)(r + l)\left(r_m - l_m + \frac{D_r - D_l}{L}\,x\right)$$

$$= RA + \tfrac{1}{6}(c_0 + c)(r + l)(r_m - l_m) + \tfrac{1}{6}(c_0 + c)(r + l)\,\frac{D_r - D_l}{L}\,x$$

$$= RA + \tfrac{1}{3}A(r_m - l_m)$$

$$+ \tfrac{1}{6}\left(c_0 + c_m + \frac{D_c}{L}\,x\right)\left(r_m + l_m + \frac{D_r + D_l}{L}\,x\right)\frac{D_r - D_l}{L}\,x.$$

If the radial section ALC_1RB (Fig. 82) be revolved, about a vertical line through P, through a distance dx along the line C_1C_1'', the center of gravity will move through the distance $\frac{\rho}{R}\,dx$, and the volume described will be

$$dV = \frac{\rho}{R}\,A\,dx.$$

$$\therefore\ V = \int_{-\frac{1}{2}L}^{+\frac{1}{2}L}\Bigg[A + \tfrac{1}{3}\,\frac{A}{R}(r_m - l_m)$$

$$+ \frac{1}{6R}\left(c_0 + c_m + \frac{D_c}{L}\,x\right)\left(r_m + l_m + \frac{D_r + D_l}{L}\,x\right)\frac{D_r - D_l}{L}\,x\Bigg]dx.$$

In integrating between these limits, the terms in the final integral containing the even powers of x will cancel each other, so that,

retaining only the even powers of x after the integral sign, we have

$$V = \left(1 + \frac{r_m - l_m}{3\,R}\right)\int_{-\frac{1}{2}L}^{+\frac{1}{2}L} A\,dx$$
$$+ \frac{D_r - D_l}{6\,R\,L}\int_{-\frac{1}{2}L}^{+\frac{1}{2}L}\left[(c_0 + c_m)\frac{D_r + D_l}{L} + \frac{D_c}{L}(r_m + l_m)\right]x^2\,dx$$
$$= \left(1 + \frac{r_m - l_m}{3\,R}\right)\int_{-\frac{1}{2}L}^{+\frac{1}{2}L} A\,dx$$
$$+ \frac{D_r - D_l}{72\,R}\left[\,(c_0 + c_m)(D_r + D_l) + D_c\,(r_m + l_m)\,\right]L.$$

If P is the volume of the straight solid above the road-bed, according to the prismoidal formula, with these cross-sections and with the length L, and G is the volume of the straight grade prism with the length L,

$$\int_{-\frac{1}{2}L}^{+\frac{1}{2}L} A\,dx = P + G.$$

The curved grade prism, whose volume also equals G, must be subtracted from V to obtain the volume above the road-bed, so that the expression for this volume ($= V - G$) becomes

$$P + \frac{r_m - l_m}{3\,R}(P + G) + \frac{D_r - D_l}{72\,R}\left[(c_0 + c_m)(D_r + D_l) + D_c\,(r_m + l_m)\right]L,$$

and hence the correction, additive to the straight volume by the prismoidal volume, is

$$\frac{r_m - l_m}{3\,R}(P + G) + \frac{D_r - D_l}{72\,R}\left[(c_0 + c_m)\,(D_r + D_l) + D_c\,(r_m + l_m)\right]L.$$

Assuming that the length of the arc $C_1'C_mC_1''$ is practically equal to the chord $C_1'C_1''$, and that the second term is negligible, the correction to the straight volume determined by the prismoidal formula is

$$\textbf{Corr.} = \frac{(r' + r'') - (l' + l'')}{6\,R}\,(P + G) \quad . \quad . \quad . \quad (16)$$

added algebraically, provided that r is outside the curve and l is inside.*

NOTE.— The point A may coincide with the point L, and B with R (Fig. 83).

* See COMPUTATION OF EARTHWORK, by JOHN WARNER, page 284 *et seq.*

136. Special Case. — *When the cross-section of the surface is a single straight line.* — (1) If, in Fig. 83, ACB is a single straight line, the correction will still be given by Eq. (16).

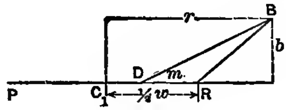

Fig. 84.

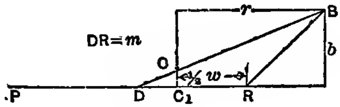

Fig. 85.

(2) When both end sections are like either Fig. 84 or Fig. 85, so that the corresponding straight solid would be like Fig. 86, the correction is

$$\frac{2w + r' + r'' - (m' + m'')}{6R} P,$$

added algebraically when B is outside the curve, and subtracted when it is inside.

Note. — The triangle $D'B'R'$ may reduce to a straight line, either $D'R'$ or $R'B'$, or to a point R'.

137. Special Case. — When both end sections are like Fig. 87,

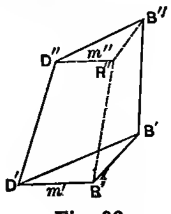

Fig. 86.

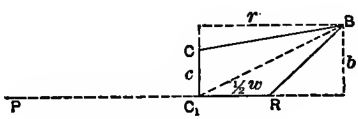

Fig. 87.

so that the corresponding straight solid would be like Fig. 88, the correction is

$$\frac{r' + r''}{6R} P + \frac{r' + r'' - w}{6R} \cdot \tfrac{1}{2} G,$$

added algebraically when B is outside the curve, and subtracted when it is inside.

Note. — Either end may be a triangle, B coinciding with R, or a straight line, CB coinciding with C_1R.

138. Special Case. — When both end sections are like Fig. 89, so that the corresponding straight solid would be like Fig. 90, the correction is

$$\frac{m' + m''}{6\,R}\,P,$$

added algebraically when B is outside the curve, and subtracted when it is inside.

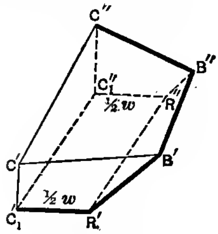

Fig. 88.

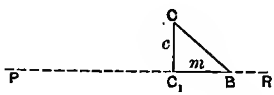

Fig. 89.

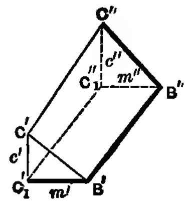

Fig. 90.

Note 1. — The triangle $C_1'C'B'$ may reduce to a straight line, either $C_1'C'$ or $C_1'B'$, or to a point C_1'.

General Note. — When the cross-section is irregular, an approximate value of the correction may be determined by first finding, at each end, the dimensions of an equivalent two-level or three-level section, by the methods given in Appendix II, and then using these equivalent sections in Eq. (16).

CHAPTER VI.

THE CROCKETT VOLUME SLIDE RULE.

139. Application. — This instrument * enables the engineer to determine mechanically the value of any one of the expressions in Appendix III, for all values of the factors and without any danger of confusion in placing the decimal point, the error in the result not exceeding perhaps 1 in 400 or 1 in 500.

140. Description. — The instrument consists of a cardboard disk, fifteen (15) inches in diameter, called the Base, and, concentric with the Base, a second cardboard disk, twelve and one-half ($12\frac{1}{2}$) inches in diameter, called the Rotator, which may be rotated about an axis through the centers of the two disks. A strip of transparent celluloid, rotating around the same axis, extends beyond the scales on the Base, and is marked with a radial line called the Mark, which enables the user to note the points, on the scales, which lie in the same radial line.

The Base bears two closed circular logarithmic scales; on the outer, called S, the numbers range from 4 to 400; and on the inner, called A, they vary from 1 to 10,000.

The Rotator bears three closed circular logarithmic scales, called B, E and P, on which the numbers range from 0.1 to 1000. P is used in connection with the prismoidal formula, while E is more closely connected with the average end area method.

When the number to be found on S is less than 4, multiply it by 100; and if it is greater than 400, divide it by 100. When the number to be found on A is less than 1, multiply it by 10,000; and if it is greater than 10,000, divide it by 10,000. When the number to be found on B, E or P is less than 0.1, multiply it by 10,000; and if it is greater than 1000, divide it by 10,000.

When the value of the expression, found on A, should be less than 1, divide the reading on A by 10,000; when it should exceed 10,000, multiply the reading by 10,000. When the value of the expression, read on B, E or

* Manufactured and for sale by W. & L. E. Gurley, Troy, N. Y.

P, should be less than 0.1, divide the reading by 10,000; when it should be greater than 1000, multiply the reading by 10,000.

On scale A, the distances from the initial point, marked 10,000, to the different numbers are proportional to the logarithms of these numbers, that is, to $\log x$. On scale S, the distances are proportional to the logarithms of the squares of the numbers divided by 108, that is, to $\log (y^2 \div 108)$. The initial points of scales S and A — that is, the points at which the logarithms

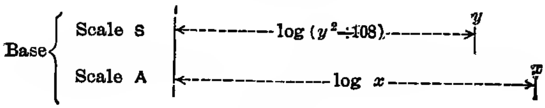

Fig. 91.

are zero — are in the same radial line, the initial point of A being in the same radial line with the number $y = \sqrt{108} = 10.4$, approximately, on S.

On scale B, the distances from the initial point, marked 1, are proportional to the logarithms of the corresponding numbers, that is, to $\log r$. On scale E, the distances are proportional to the logarithms of 108 divided by the corresponding numbers, that is, to $\log (108 \div u)$, and on P to the logarithms

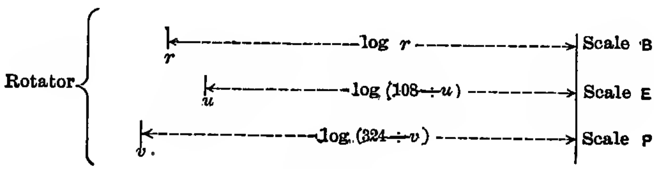

Fig. 92.

of 324 divided by the numbers, that is, to $\log (324 \div v)$. The initial points of the three scales B, E and P — that is, the points at which the logarithms are zero — are in the same radial line, so that $r = 1$ on B, $u = 108$ on E and $v = 324$ on P are in the same radial line.

On scales S and A the distances are laid off in the clockwise direction, while on B, E and P they are laid off in the counter-clockwise direction. Thus, if I represents the initial point of the scales, the arrangement of the scales on the Base is shown in Fig. 91 and that of the scales on the Rotator in Fig. 92.

Since the scales are logarithmic, the instrument performs the operations of multiplication and division by the mechanical addition and subtraction of logarithms.

APPLICATION TO CHAPTERS I, II AND III.

141. The Rules in Chapters I, II and III. — Each of the rules in these chapters, excepting Rules 9, 10(*a*), 19(*c*), 20, 21(*a*), 27(*c*), 28 and 29(*a*), after dividing by 27 to reduce to cubic yards, involves the determination of the value of the product of three factors divided by either 324 or 108, and the scales A, B, P and E were selected and arranged so as to determine these values.

142. Use of Scales A, B and E, and of A, B and P. — Scales A, B and E are used to find the value of the product of three factors divided by 108, while A, B and P are used to find the value of the product of three factors divided by 324.

For example, to find the value of $\frac{40 \times 50 \times 60}{108}$, first note, on scale A, the position of the line numbered 40; then move the Rotator until the line, on scale B, numbered 50 is in the same radial line with this 40; then, keeping the Base and the Rotator unchanged relatively, place the Mark over the line, on scale E, numbered 60; and read on scale A the number corresponding to this position of the Mark, this number $1111\frac{1}{9}$ being the value of the expression.

The word "opposite" being used as meaning "in the same radial line with," the preceding paragraph may be stated as follows:

Opposite 40 on A, set 50 on B;
Opp. 60 on E, read the value on A.

To find the value of $\frac{40 \times 50 \times 60}{324}$, use scale P instead of scale E:

Opp. 40 on A, set 50 on B;
Opp. 60 on P, read the value $370\frac{10}{27}$ on A.

When the scales A, B and E, or A, B and P, are used, the order in which the factors are taken does not affect the result; thus, *find any one of the three factors on* A, *and opposite it set either of the other two factors, found on* B; *then, opposite the third factor, found on* E *or on* P *as the case may be, read on* A *the value of the expression.*

NOTE 1. — In using the slide rule for the determination of the volume by Rule 1 or by Rule 14, the result will be more accurate if each height is diminished by the height of the lowest vertex in the corresponding end section.

NOTE 2. — In using the instrument, it may be found more convenient to place the Mark in coincidence with the first factor, found on A, then moving the Rotator until the second factor, found on B, coincides with the Mark; then bringing the Mark into coincidence with the third factor, found on E or P, and reading the result on A.

143. Selection of the Setting. — Although the result is independent of the order in which the factors are used, it will be convenient, when two factors are the same for more than one prismoid, to use these factors first. Thus, suppose we have a number of prismoids with the same length whose volumes are to be found by Rule 4. Then, in the first expression under the rule, L and $\frac{1}{2}w$ are constant, so that

Opposite $\frac{1}{2}w$ on A, set L on B;
Opp. each value of $a+b+a'+b'$ on E, read the value* on A.

In the second expression, L and $l + r$ are the same for two adjacent prismoids of the same length, so that

Opposite $l + r$ on A, set L on B;
Opp. each value of $2c + c'$ on P, read the value* on A.

144. When the Expression Contains the Square of a Factor, it is necessary to use the scale S, the setting depending upon the form of the expression. We shall consider these settings in detail.

145. The Grade Prism G. — Since the length L may differ for different prismoids,

Opp. w on S, set s on E; Opp. L on E, read G on A,
or Opp. w on S, set $2s$ on E; Opp. L on E, read $\frac{1}{2}G$ on A,
or Opp. w on S, set $4s$ on E; Opp. L on E, read $\frac{1}{4}G$ on A.

146. Rule 19(c). — The method of finding the value† of

$$y = \tfrac{1}{108} L (c + c_0)^2 (2s)$$

is dependent upon the nature of the work. If a number of prismoids have the same value of L,

Opp. L on P, set $1 \div 2s$ on A;
Opp. each value of $c + c_0$ on S, read y on P.

* In cubic yards. † See also Art. 150.

If the values of L are unequal,

Opp. $c + c_0$ on S, set 216 s on B;
Opp. each value of L on E, read y on A.

147. Rule 27(c). — To find the value * of

$$y = \frac{1}{324} L (c - c')^2 (2s),$$

when a number of prismoids have the same length L,

Opp. L on P, set $1 \div 2s$ on A;
Opp. each value of $c - c'$ on S, read y on E.

If the values of L are unequal,

Opp. $c - c'$ on S, set 216 s on B;
Opp. each value of L on P, read y on A.

148. Rules 9, 10(a), 28 and 29(a). — These rules require, after dividing by 27 to reduce to cubic yards, the determination of the value of the product of four factors † divided by 324, — that is, of the value of y when

$$y = \frac{abcd}{324}.$$

This is effected with the slide rule as follows:

Opp. a on A, set b on B, and bring the Mark to c on P;
bring d on B to the Mark, and
Opp. 1 on B, read y on A.

149. Rules 20 and 21(a). — These rules require, after dividing by 27 to reduce to cubic yards, the determination of the value of the product of four factors ‡ divided by 108, — that is, of the value of y when

$$y = \frac{abcd}{108}.$$

This is effected with the slide rule as follows:

Opp. a on A, set b on B, and bring the Mark to c on E;
bring d on B to the Mark, and
Opp. 1 on B, read y on A.

* See also Art. 150.

† By mentally multiplying one of the factors by $s_1 - s$, we may combine two of the factors, and solve with one setting, using A, B and P.

‡ By mentally multiplying one of the factors by $s_1 - s$, we may combine two of the factors, and solve with one setting, using A, B and E.

150. Level Sections. — A single setting of the Crockett Volume Slide Rule is equivalent to a table of volumes for level sections by the average end area method, and the same setting will enable the computer to determine the correction in order to find the volume according to the prismoidal formula. This setting is as follows:

Opposite 5 s' on A, set L on P; then

opposite $c + c_0$ on S, read $\frac{1}{10} E_1$ on P,
or opposite $10 (c + c_0)$ on S, read $10 E_1$ on P;

opposite $c' + c_0$ on S, read $\frac{1}{10} E_2$ on P,
or opposite $10 (c' + c_0)$ on S, read $10 E_2$ on P;

opposite c_0 on S, read $\frac{1}{20} G$ on P,
or opposite $10 c_0$ on S, read $5 G$ on P.

opposite $c - c'$ on S, read $\frac{1}{10} C$ on E,
or opposite $10 (c - c')$ on S, read $10 C$ on E.

Then the volume between the two sections is, by the average end area method,

$$E = E_1 + E_2 - G;$$

and, by the prismoidal formula,

$$P = E - C.$$

NOTE. — Arts. 142, 145, 146, 147, 148, 149 and 150 include all the settings that are needed in determining the volumes discussed in Chapters I, II and III.

151. Example for Level Sections. — If $w = 18, s = 1\frac{1}{2}, L = 60$, $c = 34$ and $c' = 64$, the volume of the solid is found as follows:

$$5 s' = 3\tfrac{1}{3}; \quad c_0 = w \div 2 s = 18 \div 3 = 6;$$

$$\therefore \quad c + c_0 = 40 \; ; c' + c_0 = 70; c - c' = 30.*$$

Opposite $3\frac{1}{3}$ on A, set 60 on P;

opposite 40 on S, read on P 266.7; $\therefore E_1 = 2667$.
opposite 70 on S, read on P 816.7; $\therefore E_2 = 8167$.
opposite 60† on S, read on P 600.0; $\therefore G = 120$.
opposite 30 on S, read on E 50.0; $\therefore C = 500$.

$\therefore E = 2667 + 8167 - 120 = 10714$ cu. yds.
$P = 10714 - 500 = 10214$ cu. yds.

* Neglect the sign. † $10 c_0$.

APPLICATION TO CHAPTER IV.

152. When the Transverse Slope of the Surface is Measured, a Single Slope on Each Side of the Center, Art. 124. — Consider each side of the center separately.

I. *When the Surface Recedes from the Road-Bed.* — Compute $c_0 = \frac{1}{2} w_l s_l'$. $(V_u = 0)$.

Opposite $c_0 + c$ on S, set L on B;
Opp. $s_l' - \tan\phi$ on B, read $V_l + \frac{1}{4} G_l$ on A.

II. *When the Surface is Level.* — Compute $c_0 = \frac{1}{2} w_l s_l'$. $(V_u = 0)$.

Opposite $c_0 + c$ on S, set L on B;
Opp. s_l' on B, read $V_l + \frac{1}{4} G_l$ on A.

III. *When the Surface Approaches the Road-Bed.*

(*a*) $\tan\phi < c \div \frac{1}{2} w_l$. — Compute $c_0 = \frac{1}{2} w_l s_l'$.

Opposite $c_0 + c$ on S, set L on B;
Opp. $s_l' + \tan\phi$ on B, read $V_l + \frac{1}{4} G_l$ on A.

(*b*) $\tan\phi \geqq c \div \frac{1}{2} w_l$.

Opposite c on S, set L on B;
Opp. $\tan\phi$ on B, read V_l on A.

(*c*) $\tan\phi > c \div \frac{1}{2} w_u$. — Compute $c_0 = \frac{1}{2} w_u s_u'$.

Opposite $c_0 - c$ on S, set L on B;
Opp. $s_u' - \tan\phi$ on B, read V_1 on A.

Opposite c on S, set L on B;
Opp. $\tan\phi$ on B, read V_2 on A.

Then $V_u = V_1 + V_2 - \frac{1}{4} G_u$.

To find the values of $\frac{1}{4} G_l$ and $\frac{1}{4} G_u$:

Opp. w_l on S, set $4 s_l$ on E; Opp. L on E, read $\frac{1}{4} G_l$ on A.
Opp. w_u on S, set $4 s_u$ on E; Opp. L on E, read $\frac{1}{4} G_u$ on A.

153. The Position of the Slope Stake when a Single Slope is Measured on Each Side of the Center. — The formulas in Art. 127 may be solved by the methods indicated in Art. 154.

ADDITIONAL APPLICATIONS.

154. Proportion. — To find the value of $y = \frac{ab}{c}$.

Opposite a on A, set b on B;
Opp. c on B, read y on A.

The setting for $y = ab$ is found by placing $c = 1$, and that for $y = \frac{a}{c}$ by placing $b = 1$.

155. Continued Products. — To find the value of $y = \frac{abd}{cef}$, separate the expression into the product of several fractions, the first of which shall be $\frac{ab}{c}$, and each of the others shall have one factor in the numerator and one in the denominator, any one or more being unity. In this case

$$y = \frac{ab}{c} \cdot \frac{d}{e} \cdot \frac{1}{f}.$$

Opp. a on A, set b on B, and bring the Mark to c on B;
bring d on B to the Mark, and then bring the Mark to e on B;
bring 1 on B to the Mark, and Opp. f on B, read y on A.

156. Correction for Curvature, First Method. — (1) *Three Level Sections.* — Use the following setting:

Opp. $r + l$ on A, set $r - l$ on B; bring Mark to $c + c_0$ on P;
bring R on P to the Mark, and
Opp. L* on P, read the value on A.

Or,
Opp. $r - l$ on A, set 3 R on E;
Opp. $E_2 + \frac{1}{2} G$* on E, read the value on A.

* Opposite each value.

(2) *Special Case*, Figs. 77, 78. — Use the following setting:

Opp. m on A, set $r + w - m$ on B;
bring Mark to b on E;
bring $3R$ on E to Mark, and
Opp. L* on E, read the value on A.

Or,

Opp. $r + w - m$ on A, set $3R$ on E;
Opp. E_2* on E, read the value on A.

(3) *Special Case*, Fig. 79. — The two terms are determined separately; denoting them by y_1 and y_2, we have:

Opp. r on S, set $c + c_0$ on B; bring Mark to 1 on B;
bring $3R$ on E to Mark, and
Opp. L* on E, read y_1 on A.

Opp. $\frac{1}{2}w$ on S, set c_0 on B; bring Mark to 1 on B;
bring $3R$ on E to Mark, and
Opp. L* on E, read y_2 on A.

Or,

Opp. r on A, set $3R$ on E; Opp. $E_2 + \frac{1}{4}G$* on E, read y_1 on A.
Opp. $\frac{1}{2}w$ on A, set $3R$ on E; Opp. $\frac{1}{4}G$* on E, read y_2 on A.

Note that y_2, for a given road-bed, side slope ratio and radius, depends only upon L (or G), so that it will be the same for all the solids in which L (or G) is the same.

(4) *Special Case*, Fig. 81. — Use the following setting:

Opp. m on S, set c on B; bring Mark to 1 on B;
bring $3R$ on E to Mark, and
Opp. L* on E, read the value on A.

Or,

Opp. m on A, set $3R$ on E;
Opp. E_2* on E, read the value on A.

* Opposite each value.

157. Correction for Curvature, Second Method. — (1) *Three-Level Sections.* — Use the following setting:

Opp. $(r' + r'') - (l' + l'')$ on A, set $6R$ on E;
Opp. $P + G$ on E, read the value on A.

(2) *Special Case,* Fig. 86. — Use the following setting:

Opp. $2w + r' + r'' - (m' + m'')$ on A, set $6R$ on E;
Opp. P on E, read the value on A.

(3) *Special Case,* Fig. 88. — The two terms are computed separately; denoting their values by y_1 and y_2, we have:

Opp. $r' + r''$ on A, set $6R$ on E;
Opp. P on E, read y_1 on A.

Opp. $r' + r'' - w$ on A, set $6R$ on E;
Opp. $\frac{1}{2}G$ on E, read y_2 on A.

(4) *Special Case,* Fig. 90. — Use the following setting:

Opp. $m' + m''$ on A, set $6R$ on E;
Opp. P on E, read the value on A.

APPENDIX I.

THE HYPERBOLIC PARABOLOID AND THE GENERAL APPLICABILITY OF THE PRISMOIDAL FORMULA.

1. Definition. — The hyperbolic paraboloid is the surface described by a straight line which slides along two fixed straight lines in such a way that it is always parallel to a fixed plane. When the two fixed lines are parallel or intersect, the hyperbolic paraboloid becomes a plane.

2. The Volume of the solid whose base is a horizontal plane, whose upper surface is a hyperbolic paraboloid, whose lateral surfaces are vertical planes through the two fixed lines, and whose ends are parallel to the fixed plane, is given correctly by the prismoidal formula. There are three cases that should be considered.

3. Case I. — Using the rectangular axes OX, OY and OZ, let ZOY be the fixed plane;* AB and CD the fixed lines (AB is not in the plane ZOX); $AOJC$ and $BLMD$ the two ends, in parallel planes, the perpendicular distance OK between them being L; $AOLB$ and $CJMD$ the lateral surfaces; EF one position of the moving line, and $EHIF$ the trapezoidal section cut by a plane parallel to ZOY.

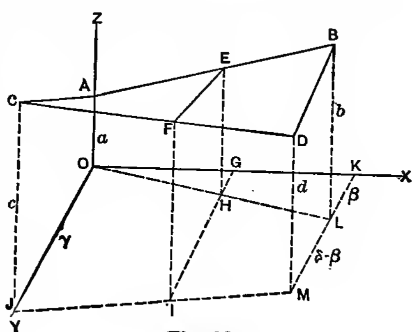

Fig. 93.

Let $OA = a$, $OK = L$, $KL = \beta$, $LB = b$, $KM = \delta$, $MD = d$, $OJ = \gamma$, $JC = c$, and $OG = x$.

* The fixed plane is here taken to be vertical, but the correct volume will be given by the prismoidal formula even though the fixed plane were inclined.

Then $HE - OA : LB - OA = OH : OL = OG : OK$

$$\therefore\ HE = a + (b - a)\frac{x}{L}.$$

Similarly, $IF = c - (c - d)\dfrac{x}{L}$,

and $HI = \gamma - (\gamma - \delta + \beta)\dfrac{x}{L}$.

We shall derive the formula for the volume below $ABDC$ first by the method of the calculus and then by the use of the prismoidal formula; the two results will be found to be identical.

(1) *By the Method of the Calculus.* — The area of $EHIF$ is

$$\tfrac{1}{2}(HE + IF)\,HI = \tfrac{1}{2}\left[(a+c) + (b-a-c+d)\frac{x}{L}\right]\left[\gamma - (\gamma-\delta+\beta)\frac{x}{L}\right]$$
$$= \tfrac{1}{2}(a+c)\gamma + \tfrac{1}{2}\left[(b-a-c+d)\gamma - (a+c)(\gamma-\delta+\beta)\right]\frac{x}{L}$$
$$- \tfrac{1}{2}(b - a - c + d)(\gamma - \delta + \beta)\frac{x^2}{L^2} \quad . \quad . \quad (a)$$

If we multiply this by dx, the result will be the volume included between the plane $EHIF$ and a parallel plane at a distance dx from it. Hence, multiplying by dx and integrating between the limits 0 and L, we have

$$\text{Volume} = \tfrac{1}{2}(a + c)\gamma L$$
$$+ \tfrac{1}{4}[(b - a - c + d)\gamma - (a + c)(\gamma - \delta + \beta)]L$$
$$- \tfrac{1}{6}(b - a - c + d)(\gamma - \delta + \beta)L,$$

which may be written

$$\text{Volume} = \tfrac{1}{12}L[2(a + c)\gamma + 2(b + d)(\delta - \beta)$$
$$+ (a + c)(\delta - \beta) + (b + d)\gamma] \quad . \quad . \quad . \quad . \quad . \quad (b)$$

(2) *By the Prismoidal Formula.* — We have

Area $AOJC = \frac{1}{2}(a + c)\gamma$; Area $BLMD = \frac{1}{2}(b + d)(\delta - \beta)$;

$$\text{Area mid-section} = \tfrac{1}{2}\left(\frac{a + b}{2} + \frac{c + d}{2}\right)\frac{\delta - \beta + \gamma}{2}.$$

$$\therefore \text{Volume} = \tfrac{1}{6}L[\tfrac{1}{2}(a + c)\gamma + \tfrac{1}{2}(b + d)(\delta - \beta)$$
$$+ \tfrac{1}{2}(a + b + c + d)(\delta - \beta + \gamma)],$$

which may be reduced to

$$\text{Volume} = \tfrac{1}{12}L[2(a + c)\gamma + 2(b + d)(\delta - \beta)$$
$$+ (a + c)(\delta - \beta) + (b + d)\gamma],$$

thus giving the same result as that found in Eq. (b) . . *Q.E.D.*

4. Case II. When the Vertical Planes $CJMD$ and $AOLB$ Intersect Each Other. — Eq. (b) will give the *difference* between the volumes above ONJ and MNL. The proof is as follows:

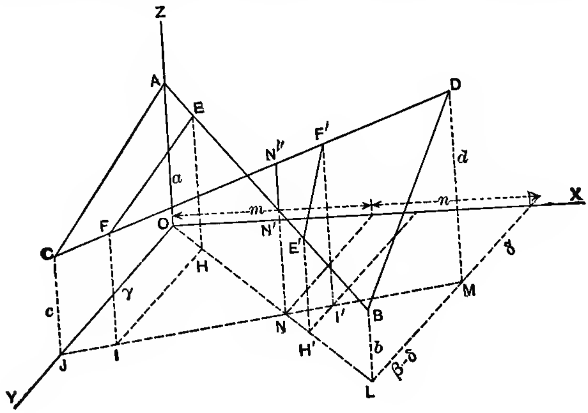

Fig. 94.

Considering the section $EHIF$ and representing its distance from ZOY by x, we have

$$HE = a - (a - b)\frac{x}{L} = a + (b - a)\frac{x}{L}, \text{ as before,}$$

and $$IF = c + (d - c)\frac{x}{L} = c - (c - d)\frac{x}{L}, \text{ as before.}$$

Since $$m : n = \gamma : \beta - \delta,$$

we have $m + n\,(= L) : m = \gamma + \beta - \delta : \gamma,$

so that $$m = \frac{L\gamma}{\gamma + \beta - \delta}.$$

Hence

$$HI = \frac{m - x}{m}\gamma = \gamma - \frac{x}{m}\gamma = \gamma - (\gamma - \delta + \beta)\frac{x}{L}, \text{ as before.}$$

Therefore the expression for the area $EHIF$ in Fig. 94 is the same as that for $EHIF$ in Fig. 93.

Considering the section $E'H'I'F'$ and representing its distance from ZOY by x, we have

$$H'E' = a + (b - a)\frac{x}{L} \quad \text{and} \quad I'F' = c - (c - d)\frac{x}{L},$$

as before; also,

$$I'H' = \frac{x - m}{m}\gamma = -\gamma + (\gamma - \delta + \beta)\frac{x}{L},$$

or

$$H'I' = \gamma - (\gamma - \delta + \beta)\frac{x}{L},$$

as before.*

Hence the expressions for the areas $EHIF$ and $E'H'I'F'$,

$$\tfrac{1}{2}(HE + IF)\,HI \text{ and } \tfrac{1}{2}(H'E' + I'F')\,H'I'$$

are alike in form, being exactly the same as Eq. (a). But the area of the cross-section on the right of N, as given by Eq. (a), will be negative; so that, when Eq. (a) is multiplied by dx and the elementary volumes summed by integration, resulting in

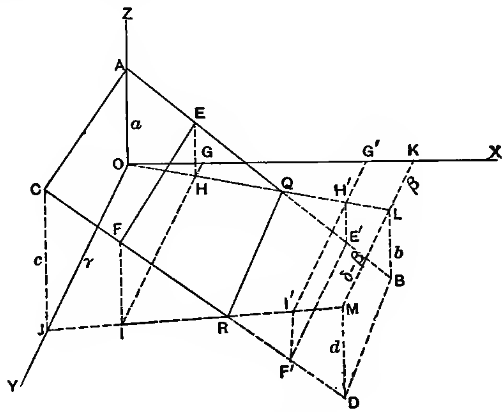

Fig. 95.

Eq. (b), we obtain the algebraic sum of the positive volume above ONJ and the negative volume above MNL, — that is, the numerical difference of the two volumes. The result given by Eq. (b) will be negative when the volume above MNL exceeds that above ONJ.

5. Case III. When the Surface *ABDC* Intersects the Plane *XOY*. — Eq. (b) will give the difference between the volume

* $H'I'$ is measured in the direction opposite to that of $I'H'$, — that is, $H'I'$ is a negative quantity. Note that $(\gamma - \delta + \beta)\frac{x}{L}$ exceeds γ when $x > m$.

above $OQRJ$ (Fig. 95) and that below $RQLM$, if b and d are considered negative, for it may be shown that Eq. (a) will give the area of any section $EHIF$ or $E'H'I'F'$, the latter area being negative since both $I'F'$ and $H'E'$ are negative.

This difficulty is avoided in practice by taking the plane XOY either entirely above or entirely below the surface $ABDC$.

6. General Applicability of the Prismoidal Formula. — If, in Eq. (a), we place

$$\tfrac{1}{2}(a+c)\gamma = A,\ \tfrac{1}{2}[(b-a-c+d)\gamma-(a+c)(\gamma-\delta+\beta)]\frac{1}{L} = B,$$

$$-\tfrac{1}{2}(b-a-c+d)(\gamma-\delta+\beta)\frac{1}{L^2} = C,$$

where A, B and C are constants, the expression for the area of the cross-section $EHIF$ becomes

$$\text{Area} = A + Bx + Cx^2,$$

and the prismoidal formula gives the true volume.

It may be demonstrated that the prismoidal formula will give the true volume of any solid with parallel ends (either end may be a line or a point) provided that the area of every parallel cross-section is expressible in the form

$$\boldsymbol{Area = A + Bx + Cx^2 + Dx^3}$$

where A, B, C and D are constant quantities.*

Hence the formula is applicable to the sphere, the ellipsoid with three unequal axes, the oblique elliptic cone, and other solids.

* One or more of the constants may be zero.

APPENDIX II.

APPROXIMATE PRISMOIDAL CORRECTION IN RAILROAD WORK.

1. When the Longitudinal Surface Lines are not Noted in the Field, the rules in Chapter III are not applicable, and the true prismoidal correction cannot be found. Some of the various methods suggested for the determination of an approximate value of this correction are given in this Appendix.

2. Method of Equivalent Level Sections. — The center heights c_L and c_L' of the level sections equivalent in area to the end sections respectively are computed, and the prismoidal correction is determined by Rule 27. (See Art. 9 of this Appendix).

3. Method of Center Heights. — The correction is computed by Rule 27, assuming that the observed center heights are the center heights of level sections approximating to the true sections, but not necessarily exactly equivalent thereto.

4. Method of Center Heights and Total Widths. — In this method, for the purpose of computing the correction, the top of the cross-section is considered as consisting of two straight lines AC and BC, the intermediate points F and G (Fig. 47) being omitted, and the correction is determined by Rule 25 for three-level sections with two warped surfaces.

5. Method of Equivalent Sections, Each with One Transverse Slope. — In Fig. 96, let $ADCEB$ be the surface of the ground, and let the area $ADCEBQC_1PA$ be known; we wish to draw a line A_eB_e such that the area of $A_eB_eQC_1P$ shall equal that of the irregular cross-section. Assuming the point A_e on the side slope, at the elevation h above the road-bed (A_e may coincide with A), draw the horizontal line A_eM and compute the area of the level section A_eMQP; subtract this area from the area of the section $ADCEBQP$, and the remainder will be the area of the triangle A_eMB_e. Divide this remainder by $\frac{1}{2}\,A_eM$ and the quotient will be NB_e. Then

$k = h + NB_e$; the values of $l_e = A_1V$ and $r_e = B_1V$ may be computed; and then that of c^e found by using the similar triangles whose hypotenuses are A_eC_e and A_eB_e.

Having made these computations, the approximate correction may be determined by Rule 24, treating the surface as a single warped surface with no longitudinal edges other than the extreme edges, or by Rule 25, considering the surface as formed by two warped surfaces.

NOTE. — If the area A_eMQP exceeds $ADCEBQP$, the point B_e will be below N.

6. Method of Equivalent Three-Level Sections. — Assume a center height C_1C_e (Fig. 96), and by a method similar to that of the preceding article determine the height k of a point B_e such that the area $C_1C_eB_eQ$ shall be equivalent to C_1CEBQ, and also the height h of a point A_e such that the area $C_1C_eA_eP$ shall be equivalent to C_1CDAP. The correction is then found by Rule 25.

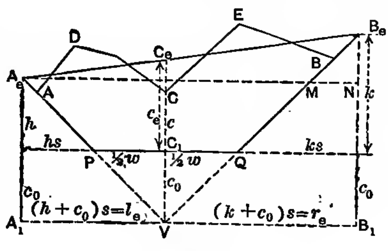

Fig. 96.

NOTE. — In the preceding articles the side slopes AP and BQ do not necessarily meet on the vertical line C_1C.

WHEN THE GRADE PRISM MAY BE USED.

7. Method of Equivalent Sections, Each with One Transverse Slope. — In Fig. 96 assume the point A_e on the side slope PA at the height h, and draw the line A_eB_e so that the area of $A_eB_eQC_1P$ shall equal that of the irregular cross-section. Then the area of the triangle A_eVB_e is

$$\begin{aligned} &\tfrac{1}{2}(A_eA_1 + B_eB_1)\,A_1B_1 - \tfrac{1}{2}A_eA_1 \,.\, A_1V - \tfrac{1}{2}B_eB_1 \,.\, B_1V \\ &= \tfrac{1}{2}(h + c_0 + k + c_0)(h + c_0 + k + c_0)\,s \\ &\qquad - \tfrac{1}{2}(h + c_0)^2 s - \tfrac{1}{2}(k + c_0)^2 s \\ &= (h + c_0)(k + c_0)\,s \,. \quad \ldots \ldots \quad (a) \end{aligned}$$

Let the area $ADCEBQC_1P$ be represented by A, and that of the grade triangle PVQ by T. Then, the value of h being assumed, we have

$$l_e = \tfrac{1}{2} w + hs \quad \text{. . . . .} \quad (b)$$

Also $$A + T = (h + c_0)(k + c_0)s = (k + c_0) l_e.$$

$$\therefore k = \frac{A + T}{l_e} - c_0 \quad \text{. . . . .} \quad (c)$$

$$r_e = \tfrac{1}{2} w + ks \quad \text{. . . . .} \quad (d)$$

Also $$A + T = \tfrac{1}{2} VC_e (l_e + r_e) = \tfrac{1}{2}(c_e + c_0)(l_e + r_e).$$

$$\therefore c_e = \frac{2(A + T)}{l_e + r_e} - c_0 \quad \text{. . . . .} \quad (e)$$

If the surface is considered as a single warped surface, the extreme edges being the only longitudinal surface edges, the correction is, by Rule 23,

$$C = \tfrac{1}{12} L (w - w')(h - h' + k - k') + \tfrac{1}{6} L (h - h')(k - k')s.$$

If the surface is considered as formed by two warped surfaces, the correction is, by Art. 109,

$$C = \tfrac{1}{12} L (\tfrac{1}{2} w - \tfrac{1}{2} w')(h - h' + k - k')$$
$$+ \tfrac{1}{12} L (c_e - c_e')[l_e + r_e - (l_e' + r_e')].$$

8. Method of Equivalent Three-Level Sections. — In Fig. 96 assume any point C_e on VC (C_e may coincide with C), and draw the line C_eA_e so that the area $C_eC_1PA_e$ shall equal the area CC_1PAD, and also draw C_eB_e so that the area $C_eC_1QB_e$ shall equal the area CC_1QBE. Denoting the area CC_1PAD by A_l and CC_1QBE by A_r and $PVC_1 = QVC_1$ by $\tfrac{1}{2} T$, we have

$$A_l + \tfrac{1}{2} T = \tfrac{1}{2}(c_e + c_0) l_e.$$

$$\therefore l = \frac{2(A_l + \tfrac{1}{2} T)}{c_e + c_0} \quad \text{. . . . .} \quad (f)$$

$$h = (l_e - \tfrac{1}{2} w) s' \quad \text{. . . . .} \quad (g)$$

Similarly

$$r_e = \frac{2\,(A_r + \frac{1}{2}\,T)}{c_e + c_0} \quad \ldots\ldots \quad (h)$$

$$k = (r_e - \tfrac{1}{2}\,w)\,s' \quad \ldots\ldots \quad (i)$$

and the correction is computed by the last formula in Art. 7 of this Appendix.

9. Equivalent Level Section. — When the grade triangle may be used, the height c_L of the level section whose area is equivalent to the area A of the given irregular cross-section is

$$c_L = \sqrt{(A + T)\,s'} - c_0$$

where $T = \frac{1}{4}\,w^2 s'$.

APPENDIX III.

SUMMARY.

The Notation is as follows: *

L = distance between the end sections;

P = volume between the end sections, by the prismoidal formula; †

E = volume between the end sections, by the average end area method; †

E_2 = volume of a right prism with the given cross-section and length $\frac{1}{2}L$; †

G = volume of the grade prism with the length L; †

$C = E - P$ or $P = E - C$. †

s = side slope ratio (horizontal ÷ vertical);

$s' = 1 \div s$;

$s_1 = \tan \theta_1$;

c_0 = height of grade prism $= \frac{1}{2}ws'$.

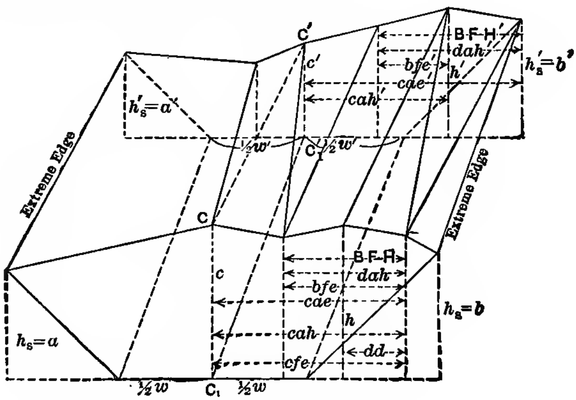

Fig. 97.

* In Fig. 97, the straight line CC' may or may not be a longitudinal surface edge.

† P, E, G and C are in cubic feet except in this Appendix where they are in cubic yards.

Measurements in one end section are represented by unprimed letters, the same letters primed representing the same measurements in the other end section, as follows:

w = width of the road-bed;

h = height at the end of any longitudinal edge (except the "extreme edges");

h_s = height at the end of an extreme edge;

bfe = horizontal distance between the back and forward *edges* (in the end section), negative when f is left of b;

cae = distance out from the center to the *edge* adjacent to the extreme edge, negative if the end of this adjacent edge and that of the extreme edge are on opposite sides of the center of the road-bed;

dah = horizontal distance between the adjacent *heights* (not necessarily at the ends of the adjacent *edges*);

dd' = greatest horizontal distance, at the other end, between the edges (if more than one) through a height h;

cah = distance out from the center to the *height* adjacent to the extreme height (not necessarily at the end of the adjacent edge), negative when this height and the extreme height are on opposite sides of the center of the road-bed;

cfe' = distance out, at the other end, from the center to the *edge* through the extreme height h_s and farthest from the extreme edge, negative when the end of this edge and that of the extreme edge are on opposite sides of the center of the road-bed;

BFH = horizontal distance between the back and forward heights, in the average end area method, negative when the forward height is to the left of the back height;

a, b, c = side heights and center height;

l, r = distances out to a and b;

d, e = heights over the edges of the road-bed;

Σdsh = sum of the two side heights at the ends of the diagonal edges (in both end sections);

Σddo = sum of the distances out to the ends of the diagonal edges (in one end section);

Σh_3 = sum of the vertical edges common to three prisms;

dh = height at the end of a diagonal surface edge;

R = radius of the curve.

I. VOLUME BY THE PRISMOIDAL FORMULA.

Rule 1. The General Solid. Fig. 17.

Clockwise; for each edge,

$\frac{1}{324} L\,(2\,h + h')\cdot bfe$ (– when f is left of b).

$\frac{1}{324} L\,(2\,h' + h)\cdot bfe'$ (– when f' is left of b').

Rule 2. The General Railroad Solid. Fig. 20.

For each edge, except the extreme edges,

$\frac{1}{324} L\,(2\,h + h')\cdot bfe,$

$\frac{1}{324} L\,(2\,h' + h)\cdot bfe'.$

For each extreme edge,

$\frac{1}{324} L\,(2\,h_s + h_s')\,(\frac{1}{2}\,w - cae)$ (– when $cae > \frac{1}{2}\,w$).

$\frac{1}{324} L\,(2\,h_s' + h_s)\,(\frac{1}{2}\,w' - cae')$ (– when $cae' > \frac{1}{2}\,w'$).

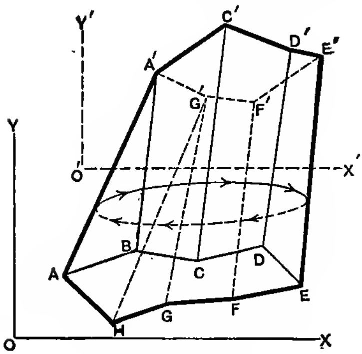

Fig. 17.

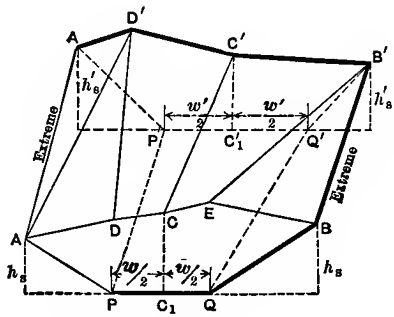

Fig. 20.

Rule 3. The Railroad Solid with Triangular Surfaces and Plane Side Slopes. Fig. 23.

For each height, except the extreme heights,

$\frac{1}{324} L\,(2\,h)\,(dah + dd').$

For each extreme height,

$\frac{1}{324} L\,(2\,h_s)\,[(\frac{1}{2}\,w - cah) + (\frac{1}{2}\,w' - cfe')^*].$

* Omit this parenthesis when no edge other than the extreme edge passes through h_s.

Rule 4. Three-Level Sections, Two Warped Surfaces. Fig. 24.

$$\tfrac{1}{108} L\,(a + b + a' + b')\,(\tfrac{1}{2} w),$$
$$\tfrac{1}{324} L\,(2\,c + c')\,(l + r),$$
$$\tfrac{1}{324} L\,(2\,c' + c)\,(l' + r').$$

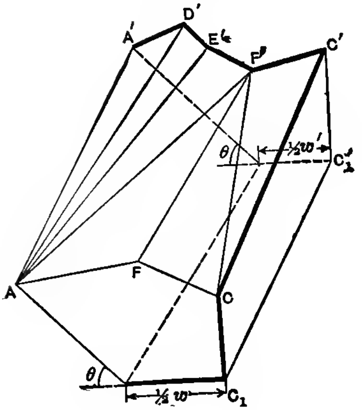

Fig. 23.

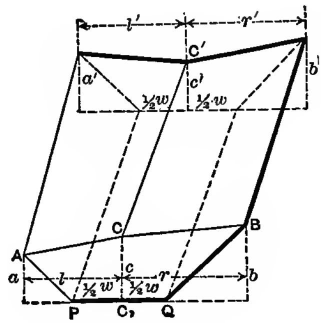

Fig. 24.

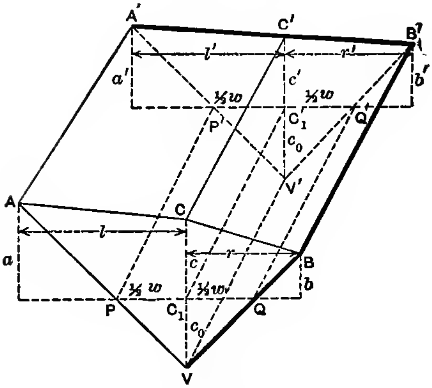

Fig. 33.

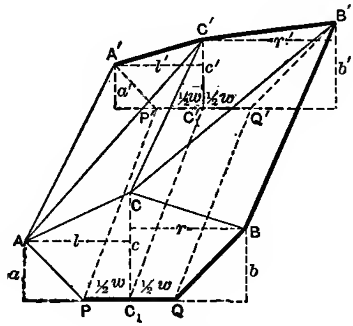

Fig. 34.

Rule 5. Three-Level Sections, Two Warped Surfaces. Fig. 33.

$$\tfrac{1}{324} L\,(2\,c + c' + 3\,c_0)\,(l + r),$$
$$\tfrac{1}{324} L\,(2\,c' + c + 3\,c_0)\,(l' + r'),$$
$$-\;G\left(= \frac{Lw^2}{108\,s}\right).$$

Rule 6. Three-Level Sections, Plane Surfaces. Fig. 34.

$$\tfrac{1}{324}\, L\, w\, (a + b + a' + b' + \Sigma\, dsh),$$

$$\tfrac{1}{324}\, L\, (2\, c)\, (l + r + \Sigma\, ddo'),$$

$$\tfrac{1}{324}\, L\, (2\, c')\, (l' + r' + \Sigma\, ddo).$$

Rule 7. Five-Level Sections, Warped Surfaces. Fig. 36.

$$\tfrac{1}{108}\, L\, (c + c')\, w,$$

$$\tfrac{1}{324}\, L\, (2\, d + d')\, l,$$

$$\tfrac{1}{324}\, L\, (2\, d' + d)\, l',$$

$$\tfrac{1}{324}\, L\, (2\, e + e')\, r,$$

$$\tfrac{1}{324}\, L\, (2\, e' + e)\, r'.$$

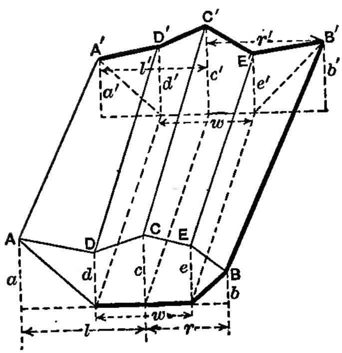

Fig. 36.

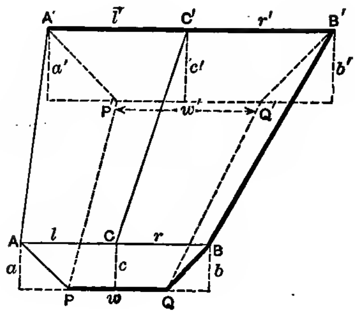

Fig. 37.

Rule 8. Level Sections. Fig. 37.

8 (a)
$$\begin{cases} \tfrac{1}{324}\, L\, (2\, c + c')\, (l + r + w), \\ \tfrac{1}{324}\, L\, (2\, c' + c)\, (l' + r' + w'). \end{cases}$$

8 (b)
$$\begin{cases} \tfrac{1}{324}\, L\, (2\, c + c')\, [2\, (w + cs)], \\ \tfrac{1}{324}\, L\, (2\, c' + c)\, [2\, (w' + c's)]. \end{cases}$$

Rule 9. When the Side Slope is Afterwards Flattened ($s_1 = \tan \theta_1$). Fig. 38.

$$\tfrac{1}{324} L\,(2\,b_1 + b_1')\,[b\,(s_1 - s)],$$

$$\tfrac{1}{324} L\,(2\,b_1' + b_1)\,[b'\,(s_1 - s)].$$

Rule 10. Sidings ($s_1 = \tan \theta_1$). Fig. 39.

10 (a)

$$\tfrac{1}{324} L\,[2\,(b + b_1) + (b' + b_1')]\,\Delta,$$

$$\tfrac{1}{324} L\,[2\,(b' + b_1') + (b + b_1)]\,\Delta',$$

$$\tfrac{1}{324} L\,(2\,b_1 + b_1')\,[b\,(s_1 - s)],$$

$$\tfrac{1}{324} L\,(2\,b_1' + b_1)\,[b'\,(s_1 - s)].$$

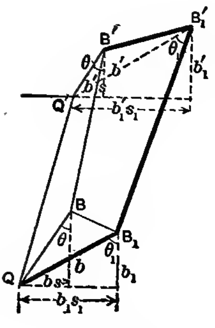

Fig. 38.

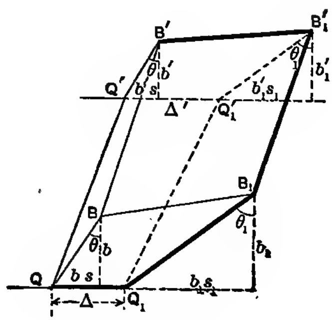

Fig. 39.

10 (b) When $s_1 = s$,

$$\tfrac{1}{324} L\,[2\,(b + b_1) + (b' + b_1')]\,\Delta,$$

$$\tfrac{1}{324} L\,[2\,(b' + b_1') + (b + b_1)]\,\Delta'.$$

10 (c) When $s_1 = s$ and $\Delta' = \Delta$,

$$\tfrac{1}{108} L\,(b + b_1 + b' + b_1')\,\Delta.$$

Rule 11. Borrow Pits.

11 (a) Triangular Bases, sides L and w. Fig. 40.

$$\tfrac{1}{324} L\,(2\,w)\,[\,\Sigma h_1 + 2\,\Sigma h_2 + 3\,\Sigma h_3 + 4\,\Sigma h_4 + 5\,\Sigma h_5 + 6\,\Sigma h_6 + 7\,\Sigma h_7 + 8\,\Sigma h_8].$$

11 (b) Rectangular Bases, sides L and w. Fig. 41.

$$\tfrac{1}{108} L w\,[\Sigma h_1 + 2\,\Sigma h_2 + 3\,\Sigma h_3 + 4\,\Sigma h_4].$$

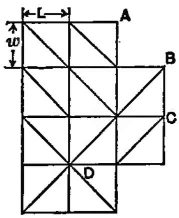

Fig. 40.

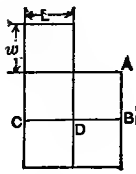

Fig. 41.

Rule 12. Side-Hill Work, — Triangular Ends, Warped Surface. Fig. 42.

$$\tfrac{1}{324} L\,(2\,h + h')\,m,$$

$$\tfrac{1}{324} L\,(2\,h' + h)\,m'.$$

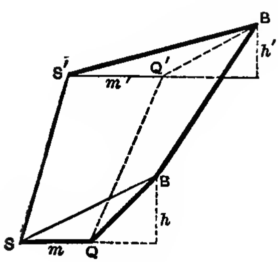

Fig. 42.

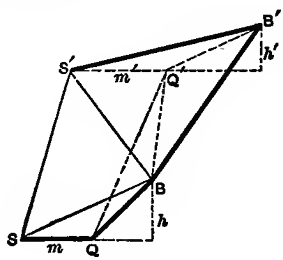

Fig. 43.

Rule 13. Side-Hill Work, — Triangular Ends, Triangular Surfaces. Fig. 43.

$$\tfrac{1}{324} L\,(2\,m)\;(h + dh'),$$

$$\tfrac{1}{324} L\,(2\,m')\;(h' + dh).$$

II. VOLUME BY THE AVERAGE END AREA METHOD.

(The volume of a right prism with the given cross-section and length $\frac{1}{2} L$.)

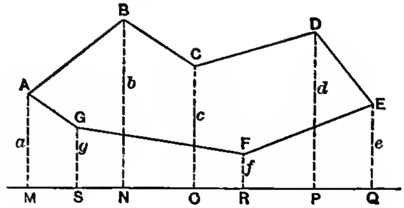

Fig. 46.

Rule 14. The General Section. Fig. 46.

Clockwise; for each height,

$$\frac{1}{108} L h \cdot BFH$$

($-$ when F is left of B).

Rule 15. The General Railroad Cross-Section. Fig. 47. For each height, except the extreme side heights,

$$\frac{1}{108} L h \cdot BFH.$$

For each extreme side height,

$$\frac{1}{108} L h_s \left(\tfrac{1}{2} w - cah\right) \qquad (- \text{ when } cah > \tfrac{1}{2} w).$$

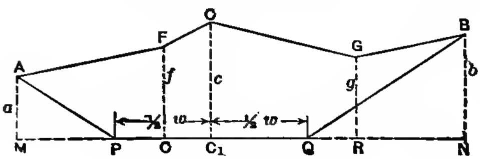

Fig. 47.

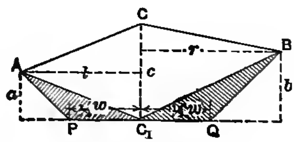

Fig. 48.

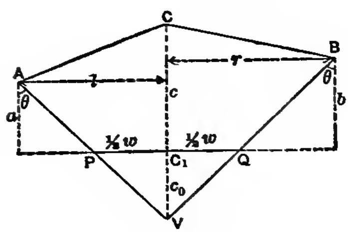

Fig. 49.

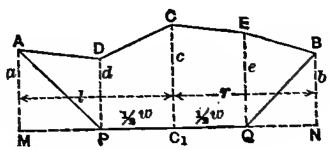

Fig. 50.

Rule 16. Three-Level Sections. Fig. 48.

$$\frac{1}{108} L \left(\tfrac{1}{2} w\right)(a + b),$$

$$\frac{1}{108} L c \,(l + r).$$

Rule 17. Three-Level Sections. Fig. 49.

$$\frac{1}{108} L (c + c_0)\,(l + r) - \tfrac{1}{2} G.$$

Rule 18. Five-Level Sections. Fig. 50.

$$\tfrac{1}{108}\, L\, d\, l,$$

$$\tfrac{1}{108}\, L\, c\, w,$$

$$\tfrac{1}{108}\, L\, e\, r.$$

Rule 19. Level Sections. Fig. 51.

19 (a) $\tfrac{1}{108}\, L\, c\, (l + r + w).$

19 (b) $\tfrac{1}{108}\, L\, c\, [2\, (w + cs)].$

19 (c) $\tfrac{1}{108}\, L\, (c + c_0)^2\, (2\, s) - \tfrac{1}{2} G.$

Rule 20. When the Side Slope is Afterwards Flattened ($\tan\theta_1 = s_1$). Fig. 52.

$$\tfrac{1}{108}\, L\, b_1\, [b\, (s_1 - s)].$$

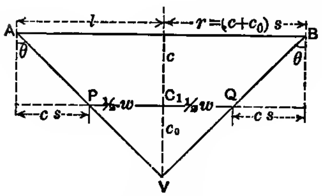

Fig. 51.

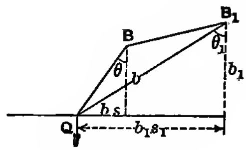

Fig. 52.

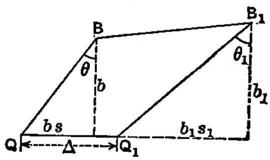

Fig. 53.

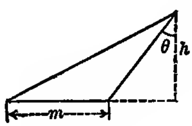

Fig. 54.

Rule 21. Sidings ($\tan\theta_1 = s_1$). Fig. 53.

21 (a) $\tfrac{1}{108}\, L\, (b + b_1)\, \Delta,$

$$\tfrac{1}{108}\, L\, b_1\, [b\, (s_1 - s)].$$

21 (b) If $s_1 = s$,

$$\tfrac{1}{108}\, L\, (b + b_1)\, \Delta.$$

Rule 22. Side-Hill Work, — Triangular End Section. Fig. 54.

$$\tfrac{1}{108}\, L\, h\, m.$$

III. THE PRISMOIDAL CORRECTION.

Rule 23. The General Solid. Fig. 17.

Clockwise; for each edge,

$$\tfrac{1}{324} L (h - h') (bfe - bfe').$$

Rule 24. The General Railroad Solid. Fig. 20.

For each edge, except the extreme edges,

$$\tfrac{1}{324} L (h - h') (bfe - bfe').$$

For each extreme edge,

$$\tfrac{1}{324} L (h_s - h_s') [(\tfrac{1}{2} w - cae) - (\tfrac{1}{2} w' - cae')].$$

Rule 25. Three-Level Sections, Two Warped Surfaces. Fig. 24.

$$\tfrac{1}{324} L (c - c') [(l + r) - (l' + r')].$$

Rule 26. Five-Level Sections, Warped Surfaces. Fig. 36.

$$\tfrac{1}{324} L (d - d') (l - l'),$$
$$\tfrac{1}{324} L (e - e') (r - r').$$

Rule 27. Level Sections. Fig. 37.

27 (a) $\tfrac{1}{324} L (c - c') [(l + r + w) - (l' + r' + w')].$

27 (b) $\tfrac{1}{324} L (c - c') [(l + r) - (l' + r')].$

27 (c) $\tfrac{1}{324} L (c - c')^2 (2\,s).$

Rule 28. When the Side Slope is Afterwards Flattened ($\tan \theta_1 = s_1$). Fig. 38.

$$\tfrac{1}{324} L (s_1 - s) (b - b') (b_1 - b_1').$$

Rule 29. Sidings ($\tan \theta_1 = s_1$). Fig. 39.

29 (a) $\tfrac{1}{324} L [(b + b_1) - (b' + b_1')] (\Delta - \Delta'),$

$\tfrac{1}{324} [L (s_1 - s)] (b - b') (b_1 - b_1').$

29 (b) When $s_1 = s$,

$\tfrac{1}{324} L [(b + b_1) - (b' + b_1')] (\Delta - \Delta').$

29 (c) When $s_1 = s$ and $\Delta' = \Delta$, $C = 0$.

Rule 30. Side-Hill Work, — Triangular Ends, Warped Surface. Fig. 42.

$$\tfrac{1}{324} L (h - h') (m - m').$$

IV. WHEN THE TRANSVERSE SLOPE OF THE SURFACE IS MEASURED.

(The volume of a right prism with the length $\frac{1}{2} L$. Consider each side of the center separately.)

1. When the Surface Recedes from the Road-Bed. Fig. 65.

$$V_l = \frac{L}{108} \frac{(c_0 + c)^2}{s_l' - \tan \phi} - \tfrac{1}{4} G_l. \quad c_0 = \frac{w_l}{2\, s_l}. \quad V_u = 0.$$

2. When the Surface is Level. Fig. 66.

$$V_l = \tfrac{1}{108} L\, (c_0 + c)^2\, s_l - \tfrac{1}{4} G_l. \quad c_0 = \frac{w_l}{2\, s_l}. \quad V_u = 0.$$

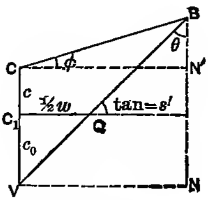

Fig. 65.

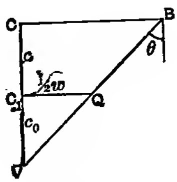

Fig. 66.

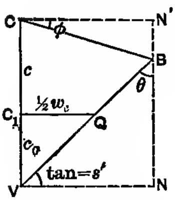

Fig. 67.

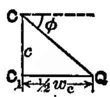

Fig. 68.

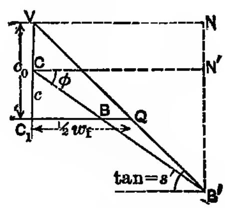

Fig. 69.

3. When the Surface Approaches the Road-Bed.

(*a*) $\tan \phi < c \div \frac{1}{2} w_l$, Fig. 67.

$$V_l = \frac{L}{108} \frac{(c_0 + c)^2}{s_l' + \tan \phi} - \tfrac{1}{4} G_l. \quad c_0 = \frac{w_l}{2\, s_l}.$$

(*b*) $\tan \phi \geqq c \div \frac{1}{2} w_l$, Figs. 68, 69.

$$V_l = \frac{L}{108} \frac{c^2}{\tan \phi}.$$

(*c*) $\tan \phi > c \div \frac{1}{2} w_u$, Fig. 69.

$$V_u = \frac{L}{108} \frac{(c_0 - c)^2}{s_u' - \tan \phi} + \frac{L}{108} \frac{c^2}{\tan \phi} - \tfrac{1}{4} G_u. \quad c_0 = \frac{w_u}{2\, s_u}.$$

V. CORRECTION FOR CURVATURE. FIRST METHOD.

(To find the total correction at any station, or at any substation, let L be the sum of the lengths of the adjacent chords.)

1. Three-Level Sections. Fig. 83.

$$\frac{L\,(r+l)\,(r-l)\,(c+c_0)}{324\,R}, \quad \text{or} \quad \frac{r-l}{3\,R}\,(E_2+\tfrac{1}{2}\,G).$$

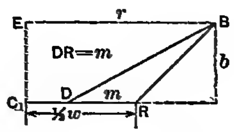

Fig. 77.

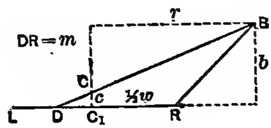

Fig. 78.

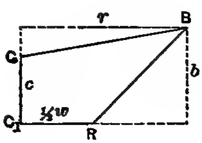

Fig. 79.

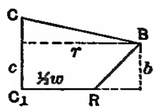

Fig. 80.

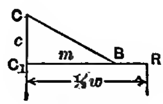

Fig. 81.

2. Special Cases.

(a) Figs. 77, 78. $\quad \dfrac{L\,b\,m(r+w-m)}{324\,R}, \quad \text{or} \quad \dfrac{r+w-m}{3\,R}\,E_2.$

(b) Figs. 79, 80. $\quad \dfrac{L\,r^2\,(c+c_0)}{324\,R} - \dfrac{L\,c_0}{324\,R}\left(\dfrac{w}{2}\right)^2,$

or $\quad \dfrac{r}{3\,R}\,(E_2+\tfrac{1}{4}\,G) - \dfrac{\frac{1}{2}\,w}{3\,R}\,(\tfrac{1}{4}\,G).$

(c) Fig. 81. $\quad \dfrac{L\,m^2\,c}{324\,R}, \quad \text{or} \quad \dfrac{m}{3\,R}\,E_2.$

VI. CORRECTION FOR CURVATURE. SECOND METHOD.

(The correction is the total correction for the solid between two adjacent end sections.)

1. Three-Level Sections. Fig. 83.

$$\frac{(r' + r'') - (l' + l'')}{6\,R}\,(P + G).$$

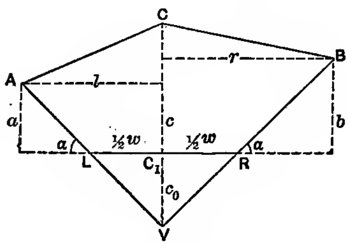

Fig. 83.

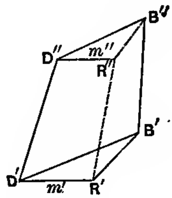

Fig. 86.

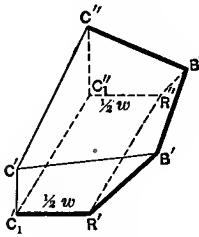

Fig. 88.

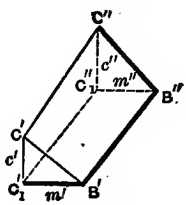

Fig. 90.

2. Special Cases.

(a) Fig. 86. $\dfrac{2\,w + r' + r'' - (m' + m'')}{6\,R}\,P.$

(b) Fig. 88. $\dfrac{r' + r''}{6\,R}\,P + \dfrac{r' + r'' - w}{6\,R}\cdot\tfrac{1}{2}\,G.$

(c) Fig. 90. $\dfrac{m' + m''}{6\,R}\,P.$

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